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Portfolio-specific mortality modelling — graduating your own tables

Stephen J. Richards
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1. About the speaker

1. About the speaker

- Consultant on longevity risk since 2005
- Founded longevity-related software businesses in 2006:



- Joint venture with Heriot-Watt in 2009:



1. About the material

- Richards, Kaufhold and Rosenbusch (2013).
- Results published in European Actuarial Journal in December 2013.
- Available online at www.springerlink.com

2. Data description

2. Data description

Multi-employer pension arrangement in Germany:

- 253,444 pension records.
- 31,842 deaths in 2007–2011.
- 1.03 million life-years lived in 2007–2011.

Source: Richards, Kaufhold and Rosenbusch (2013).

2. Data description

Unequal distribution of liabilities:

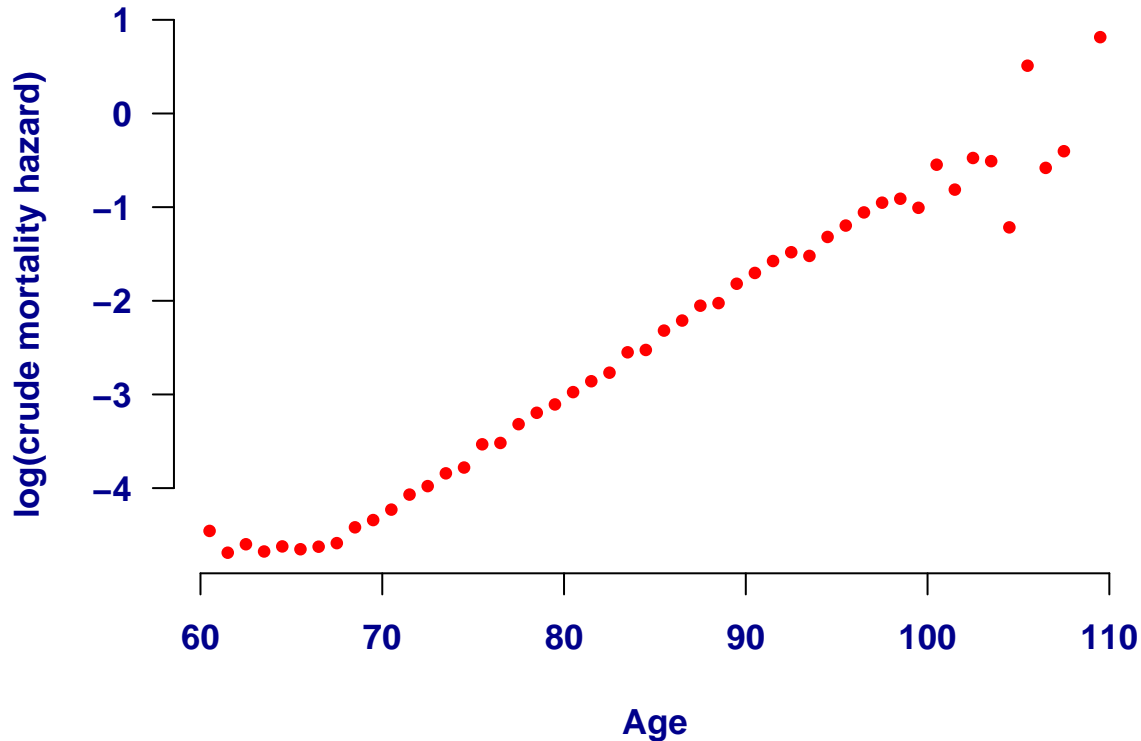
- 50% of all pensions are received by just 23.5% of lives.
- males are 34.5% of lives, but 59.7% of large-pension cases.

Source: Richards, Kaufhold and Rosenbusch (2013).

3. Exploratory data analysis

3. Exploratory data analysis

$\log_e(\text{crude mortality hazard})$ from age 60, males and females combined:



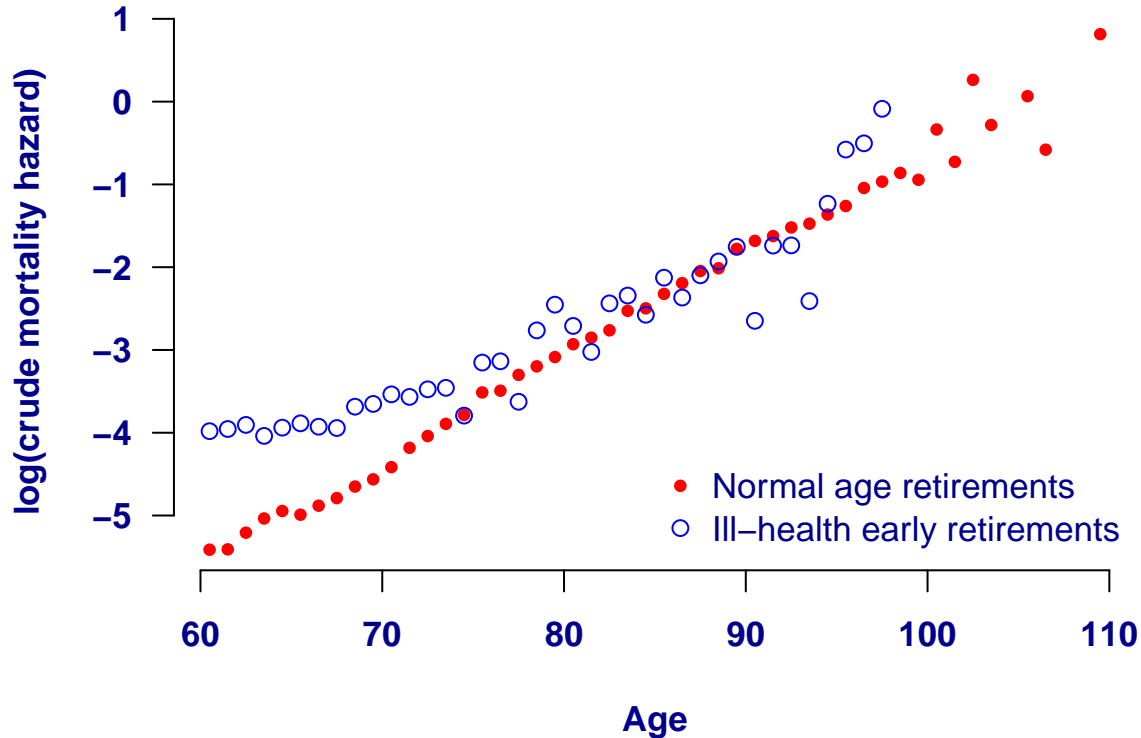
Source: Richards, Kaufhold and Rosenbusch (2013), Figure 1.

3. Exploratory data analysis

- Mortality increases with age.
- Smoothing is needed to iron out random variation.
- Extrapolation is needed for highest ages.

3. Exploratory data analysis

$\log_e(\text{crude mortality hazard})$ from age 60 by retirement type:



Source: Richards, Kaufhold and Rosenbusch (2013), Figure 4.

3. Exploratory data analysis

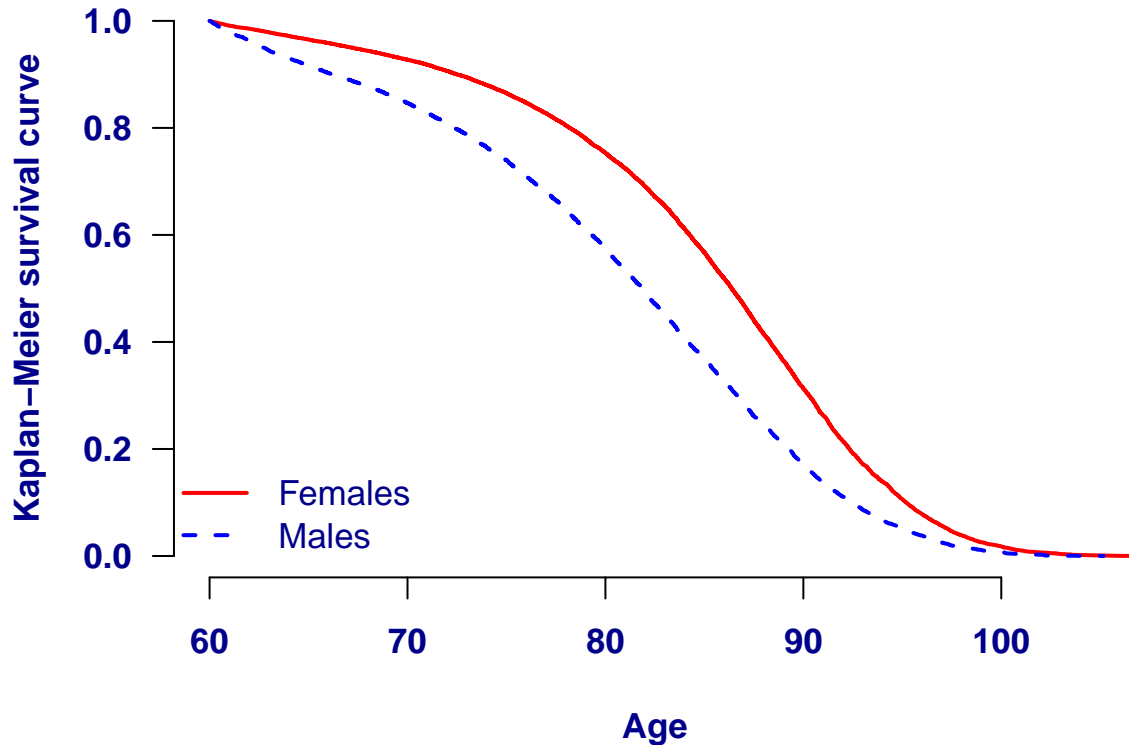
- Strong excess mortality for ill-health retirals, but
- Excess ill-health mortality reduces with increasing age.
- This phenomenon is known as *mortality convergence*.

3. Exploratory data analysis

- A useful tool is the survival function from Kaplan and Meier (1958).
- This can reveal useful risk factors to consider...
- ...and it can reveal data problems an A/E comparison cannot detect.

3. Exploratory data analysis

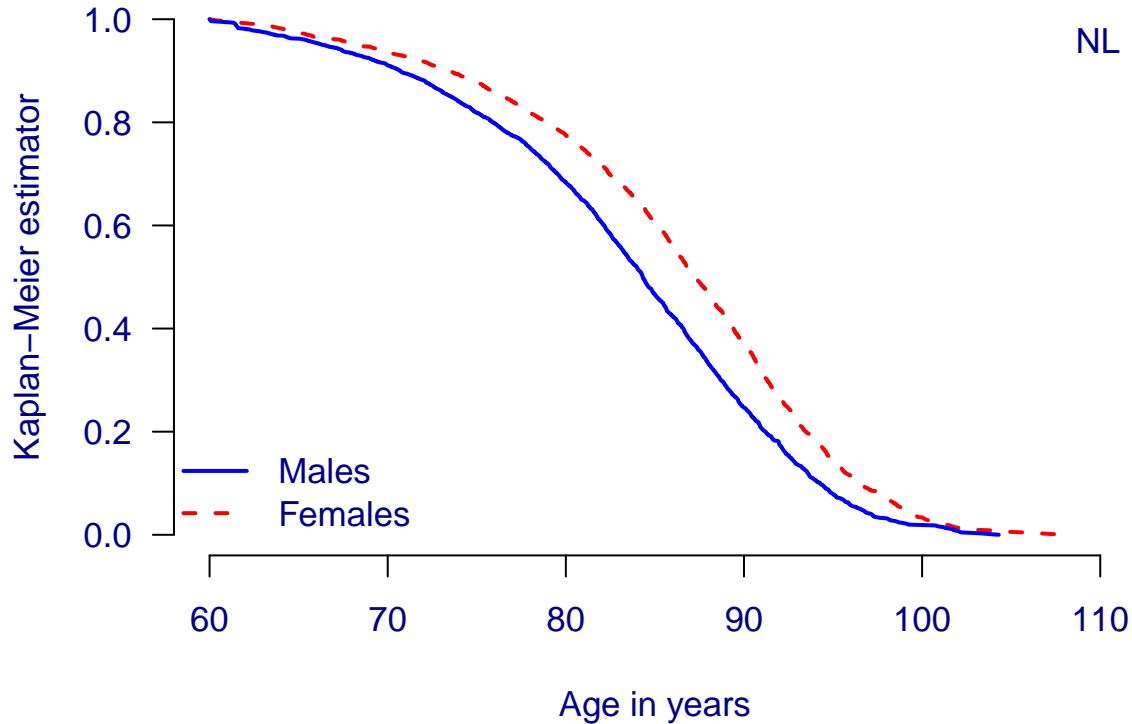
Kaplan-Meier product-limit estimator by gender from age 60:



Source: Richards, Kaufhold and Rosenbusch (2013), Figure 2.

3. Exploratory data analysis

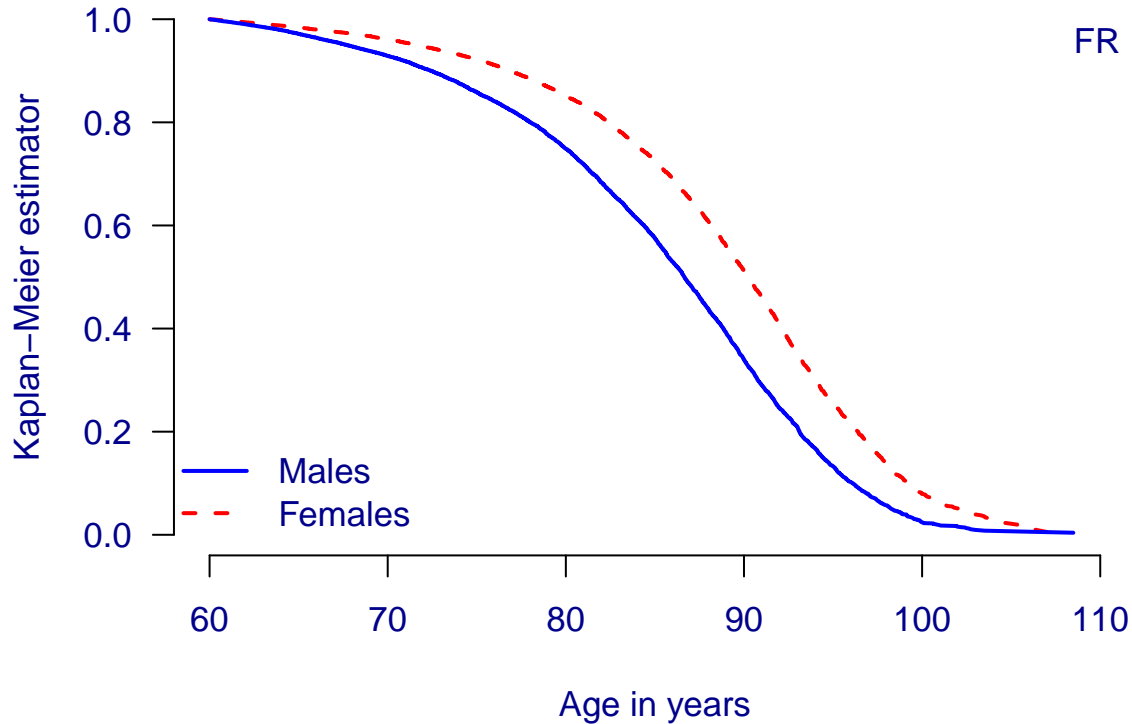
Kaplan-Meier product-limit estimator by gender from age 60:



Source: Dutch private-sector pension scheme.

3. Exploratory data analysis

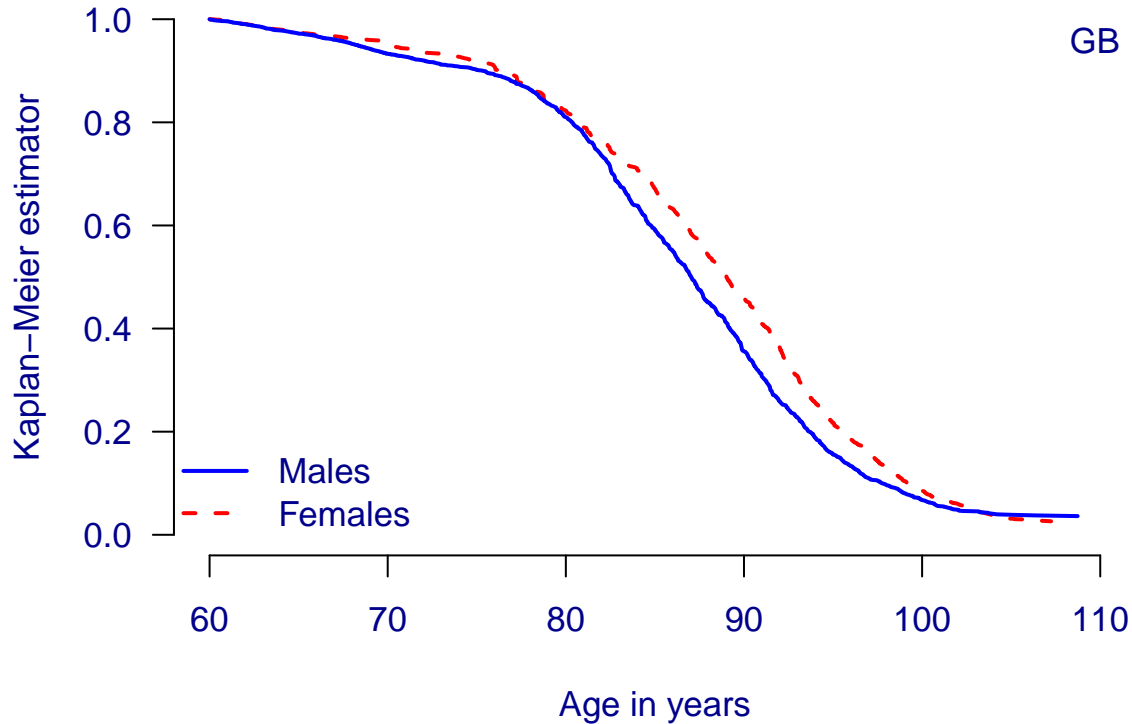
Kaplan-Meier product-limit estimator by gender from age 60:



Source: French public-sector top-up pension scheme.

3. Exploratory data analysis

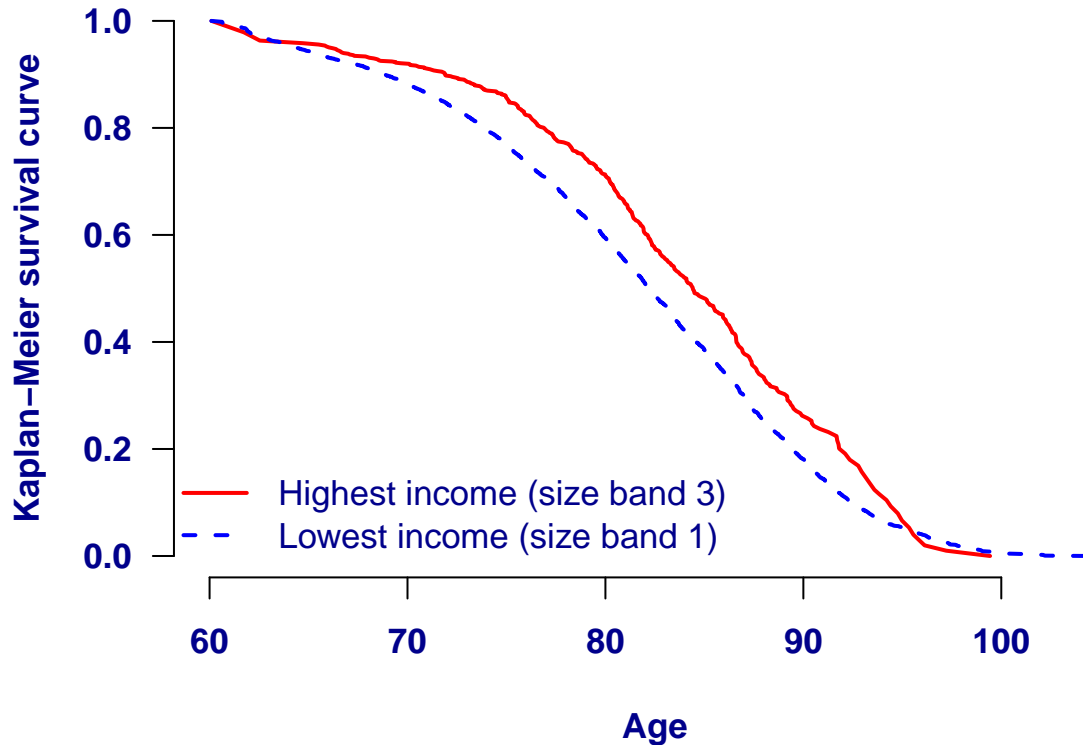
Kaplan-Meier product-limit estimator by gender from age 60:



Source: UK annuity portfolio.

3. Exploratory data analysis

Kaplan-Meier product-limit estimator by income from age 60:



Source: Richards, Kaufhold and Rosenbusch (2013), Figure 3.

3. Exploratory data analysis

The data tell us what the requirements of the model are:

- smooth out random variation,
- extrapolate to higher ages,
- allow for multiple risk factors simultaneously, and
- allow risk factors to vary their impact by age.

4. Model structure and fitting

4. Model structure

- All requirements are fulfilled by a parametric survival model.
- Here we will use the Makeham-Perks law:

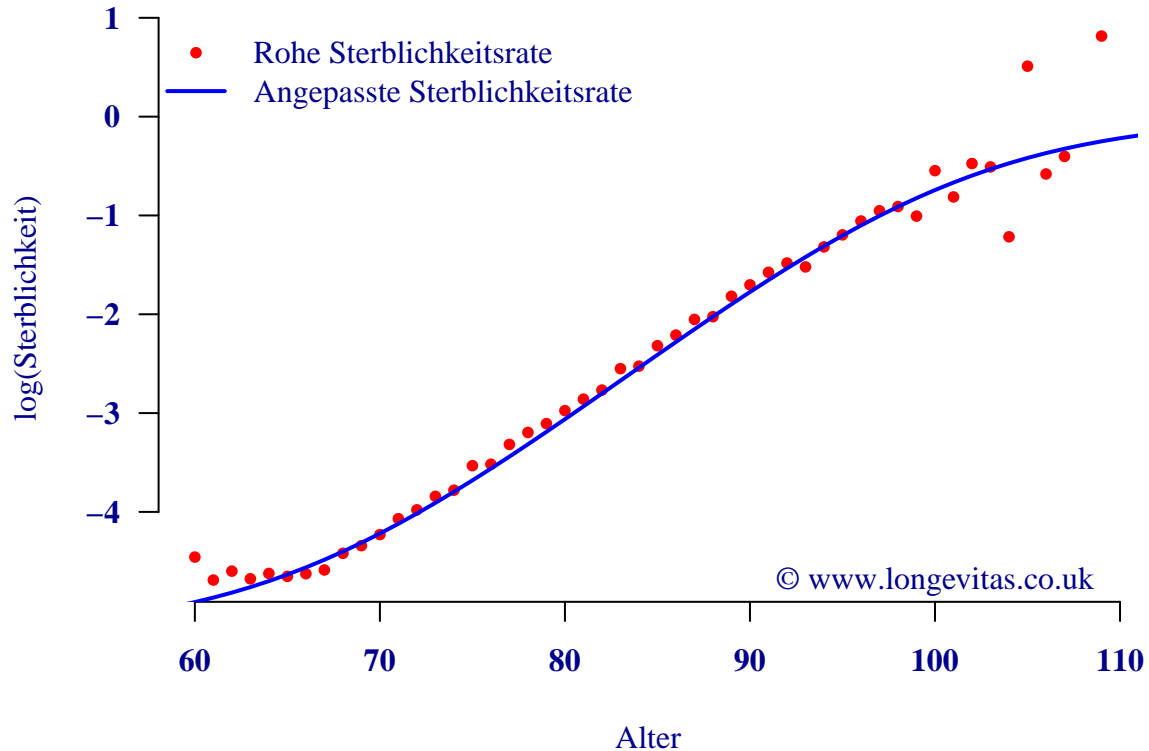
$$\mu_x = \frac{e^\epsilon + e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$$

with real-valued age x and real-valued parameters ϵ , α and β .

Source: Richards (2008, 2012).

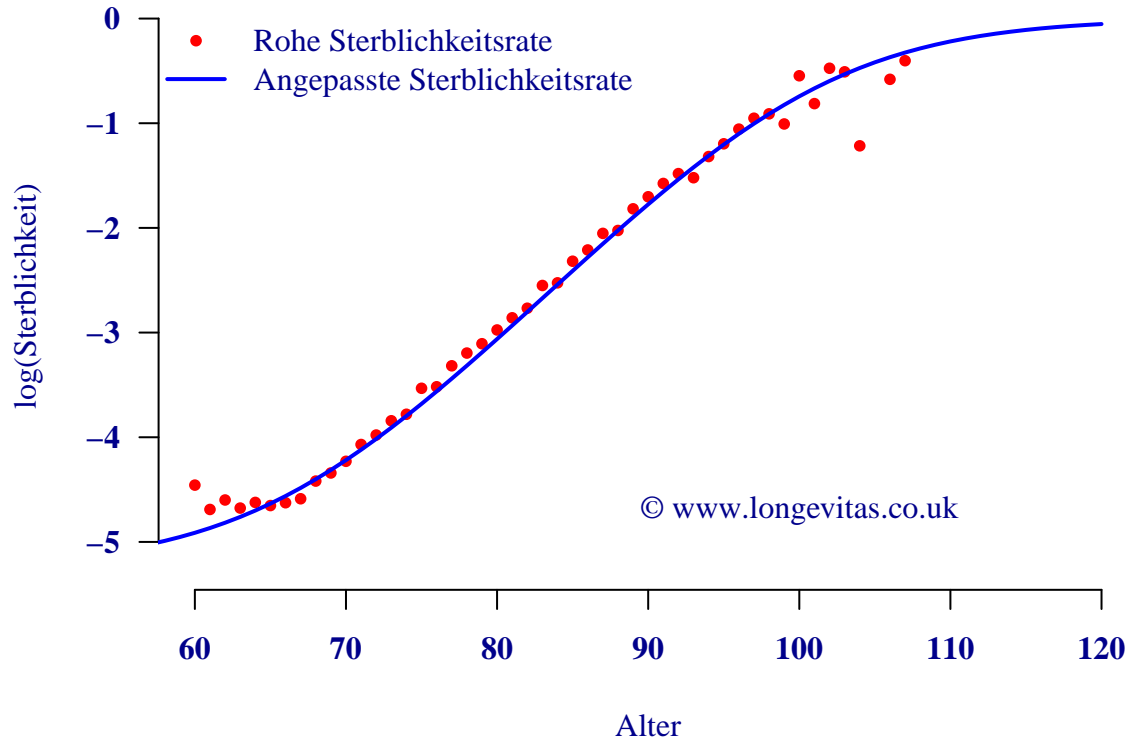
4. Model features

Automatic smoothing of random variation:



4. Model features

Sensible extrapolation to higher ages:



4. Model fitting: method of maximum likelihood

Likelihood function:

$$L = \prod_{i=1}^n t_i p_{x_i} \mu_{x_i+t_i}^{d_i}$$

where:

- x_i is the entry age for life i of n lives,
- t_i is the time observed, and
- $d_i = 1$ if life i is dead, otherwise $d_i = 0$.

4. Model structure

Simple relationship between μ_x and survival probability ${}_t p_x$:

$$\begin{aligned} {}_t p_x &= \exp \left(- \int_0^t \mu_{x+s} ds \right) \\ &= \exp (-H_x(t)) \end{aligned}$$

$H_x(t)$ is the *integrated hazard function*.

4. Model fitting: method of maximum likelihood

Optimisation is often easier with the log-likelihood function:

$$\begin{aligned}\ell &= \log L \\ &= \sum_{i=1}^n -H_{x_i}(t_i) + \sum_{i=1}^n d_i \log \mu_{x_i+t_i}\end{aligned}$$

where $H_x(t) = \int_0^t \mu_{x+s} ds$.

Richards (2012) tabulates μ_x and $H_x(t)$ for sixteen models.

4. Model structure

- Assume α should vary by gender:

$$\alpha_i = \alpha_0 + \alpha_M z_i$$

where:

- α_0 is the so-called *baseline*,
 - α_M is the effect of being male, and
 - $z_i = 1$ if life i is male, otherwise $z_i = 0$ if life i is female.
- α_M measures the mortality difference for being male.
 - Alternatively, we could set males as the baseline and estimate α_F .

4. Model structure

- Simple extension to j risk factors:

$$\alpha_i = \alpha_0 + \sum_{j=1}^m \alpha_j z_{j,i}$$

where:

- α_j is the effect of risk factor j , and
- $z_{j,i} = 1$ if life i has risk factor j , otherwise $z_{j,i} = 0$.
- $\alpha_j < 0$ when mortality is reduced, $\alpha_j > 0$ when mortality is raised.
- No minimum number of lives for estimating α_j .

5. Results

5. Results for German pensioners

Seven statistically significant risk factors for longevity:

- age,
- gender,
- pension size,
- retirement status: normal, ill-health or widow(er),
- employer type,
- region, and
- time

Source: Richards, Kaufhold and Rosenbusch (2013).

5. Results for German pensioners

Financial impact on annuity factors at age 65:

| Risk factor | Change | Annuity factor | Relative change |
|--------------------------|-------------------|-----------------------|------------------------|
| Base case | - | 16.114 | |
| Gender | Female→male | 14.529 | -9.8% |
| Retirement health status | Normal→ill-health | 12.974 | -10.7% |
| Pension size | Largest→smallest | 11.717 | -9.7% |
| Region | B→P | 11.025 | -5.9% |
| Employer type | Private→public | 10.599 | -3.9% |
| Overall | | | -34.2% |

Source: Richards, Kaufhold and Rosenbusch (2013), Appendix 1.

5. Results — international comparison

- How do these results compare with other data sets?
- Consider annuities with a UK insurer...

5. Results for UK annuitants

UK insurer with six available risk factors:

- age,
- gender,
- lifestyle (via postcode),
- duration (time since annuity purchase),
- pension size, and
- region.

Source: Richards and Jones (2004).

5. Results for UK annuitants

Financial impact of mortality rating factors:

| Factor | Step change | Reserve | Change |
|---------------|--------------------|----------------|---------------|
| Base case | - | 13.39 | |
| Gender | Female→male | 12.14 | -9.3% |
| Lifestyle | Top→bottom | 10.94 | -9.9% |
| Duration | Short→long | 9.88 | -9.7% |
| Pension size | Largest→smallest | 9.36 | -5.2% |
| Region | South→North | 8.90 | -4.9% |
| Overall | | | -33.6% |

Source: Richards and Jones (2004), page 39.

5. What risk factors should you use?

- Each portfolio is unique.
- Business practice determines available information.
- Fit models to your data using business-relevant risk factors.
- Even small portfolios can have significant characteristics of their own. . .

5. Impact of scheme-specific mortality

- Return to German pensioner data.
- The largest scheme has approximately 12,000 members.
- Do the seven risk factors explain the mortality variation in this scheme?

5. Impact of scheme-specific mortality

- Mortality around 10% lower for largest scheme.
- Effect exists *even after allowing for all seven other risk factors*.
- Result was highly statistically significant (p-value 0.0001).
- Impact was an extra $2-2\frac{1}{2}\%$ on reserves.

6. Conclusions

6. Conclusions

- Kaplan-Meier methods reveal important risk factors...
- ...and detect problems hidden from traditional A/E methods.
- A parametric survival model simultaneously:
 - identifies the main risk factors,
 - identifies any interactions with age,
 - smoothes (graduates) the rates, and
 - extrapolates to higher ages.
- Even small portfolios can have significant characteristics of their own.



References

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