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The APCI model — a stochastic implementation

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Overview

1. Background
2. APCI model
3. Fitting and constraints
4. Parameter estimates
5. Smoothing
6. Value-at-Risk (VaR)
7. Constraints (again)
8. Conclusions

1 Background

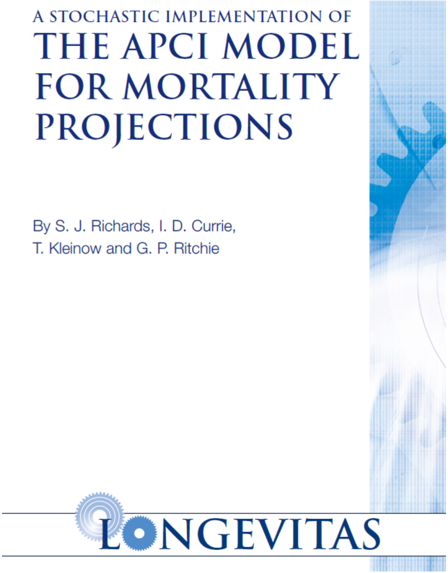
1 Background

- CMI released new projection spreadsheet.
- Calibration is done by new APCI model.
- See Continuous Mortality Investigation (2017).

1 Background

- CMI intended APCI model for calibrating spreadsheet.
- Richards et al. (2017) implement it as a fully stochastic model...

  ... to be presented at sessional meeting in 2018.


A STOCHASTIC IMPLEMENTATION OF

THE APCI MODEL FOR MORTALITY PROJECTIONS

By S. J. Richards, I. D. Currie, T. Kleinow and G. P. Ritchie

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2 APCI model

2 APCI model

$$\log m_{x,y} = \alpha_x + \beta_x(y - \bar{y}) + \kappa_y + \gamma_{y-x} \qquad (1)$$

2 Related models for $\log m_{x,y}$

$$\text{Age-Period} : \alpha_x + \kappa_y \tag{2}$$

$$\text{APC} : \alpha_x + \kappa_y + \gamma_{y-x} \tag{3}$$

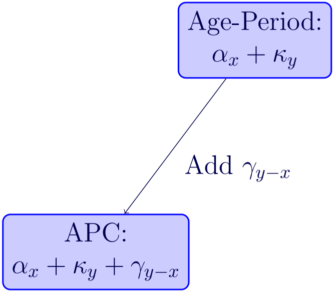
$$\text{Lee-Carter} : \alpha_x + \beta_x \kappa_y \tag{4}$$

$$\text{APCI} : \alpha_x + \beta_x (y - \bar{y}) + \kappa_y + \gamma_{y-x} \tag{5}$$

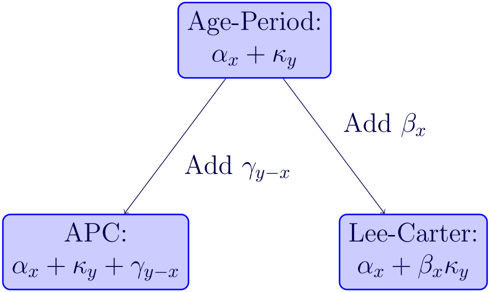
2 Model relationships

Age-Period:
$\alpha_x + \kappa_y$

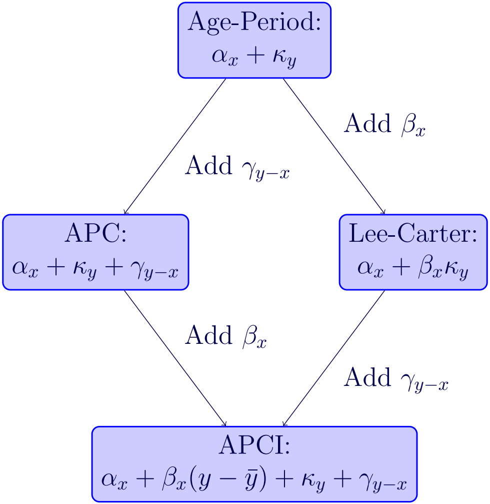
2 Model relationships



2 Model relationships



2 Model relationships



2 APCI model

APCI model can be viewed as either:

- An APC model with added Lee-Carter-like β_x term, or
- A Lee-Carter-like model with added γ_{y-x} cohort term.

BUT, κ_y in APCI model is very different, as we will see.

3 Fitting and constraints

3 Constraints required

- All of these models require *identifiability constraints.*
- Identifiability constraints do not change $\log \hat{\mu}_{x,y}$.

3 Constraints required

$$\text{AP} : \sum \kappa_y = 0 \tag{6}$$

$$\text{LC} : \sum \kappa_y = 0, \sum \beta_x = 1 \tag{7}$$

$$\text{APC} : \sum \kappa_y = 0, \sum \gamma_c = 0, \sum (c - c_{\text{min}} + 1)\gamma_c = 0 \tag{8}$$

3 Constraints required

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APCI model requires five identifiability constraints:

$$\sum \kappa_y = 0 \tag{9}$$

$$\sum (y - y_1)\kappa_y = 0 \tag{10}$$

$$\sum \gamma_c = 0 \tag{11}$$

$$\sum (c - c_{\text{min}} + 1)\gamma_c = 0 \tag{12}$$

$$\sum (c - c_{\text{min}} + 1)^2\gamma_c = 0 \tag{13}$$

3 APCI model

- APCI model requires more constraints than other models.
- Constraints impact the parameter estimates in important ways.

3 Not all cohorts are equal

- Continuous Mortality Investigation (2017) uses (for example) $\sum \gamma_c = 0$.

  $\Rightarrow$ Cohort with one observation gets same weight as cohort with thirty observations?

3 Not all cohorts are equal

- Cairns et al. (2009) weights according to number of observations, i.e. $\sum w_c \gamma_c = 0$.
- Cairns et al. (2009) approach preferable, so used from now on.
- See also Richards et al. (2017, Appendix C).

3 Fitting

The Age-Period, APC and APCI models:

- are linear,
- require identifiability constraints, and
- have parameters that can be smoothed.

3 Fitting

- Assume $D_{x,y} \sim \text{Poisson}(E_{x,y}\mu_{x,y})$.
- AP, APC and APCI models are penalized, smoothed GLMs.

3 Fitting

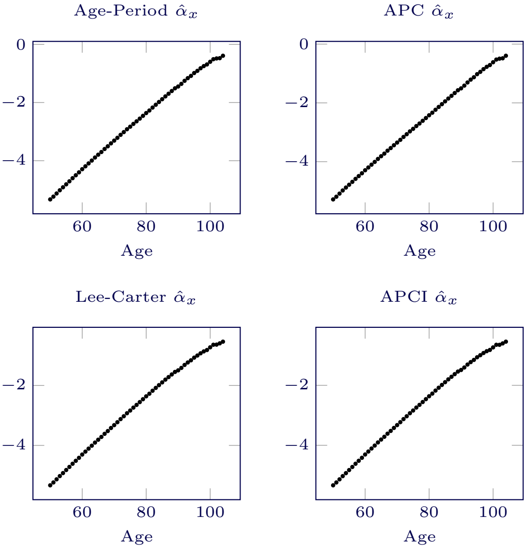
Algorithm from Currie (2013) is integrated GLM-fitting process to:

- maximise likelihood,
- apply identifiability constraints, and
- smooth parameters.

4 Parameter estimates

4 α_x

Parameter estimates $\hat{\alpha}_x$ for four unsmoothed models.

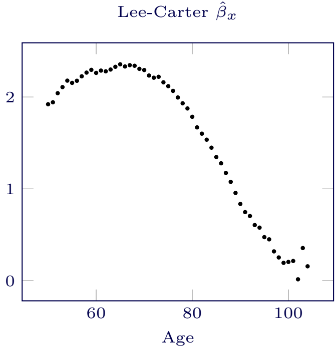


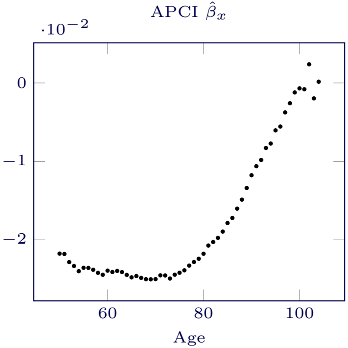
4 α_x

$\Rightarrow$ α_x plays the same role across all four models, i.e. average log mortality by age.

...as long as $\sum_y \kappa_y = 0$.

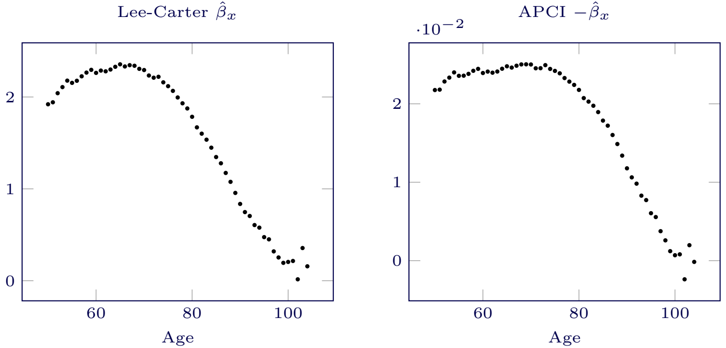
Parameter estimates $\hat{\beta}_x$ for Lee-Carter and APCI models (both unsmoothed).





4 β_x

Parameter estimates $\hat{\beta}_x$ for Lee-Carter and $-\hat{\beta}_x$ for APCI models (both unsmoothed).



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$\Rightarrow$ β_x plays an analogous role in the Lee-Carter and APCI models, namely an age-related modulation of the time index.

4 β_x

But the APCI model has *two* time indexes:

1. A modulated central linear trend, $(y - \bar{y})$, and
2. An unmodulated non-linear term, κ_y.

4 Conclusions for α_x and β_x

- α_x and β_x play similar roles across all models.
- What about κ_y and γ_{y-x}?

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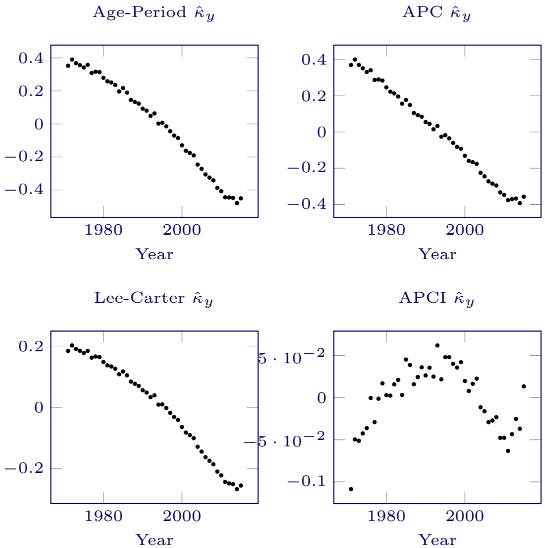
Parameter estimates $\hat{\kappa}_y$ for four unsmoothed models.

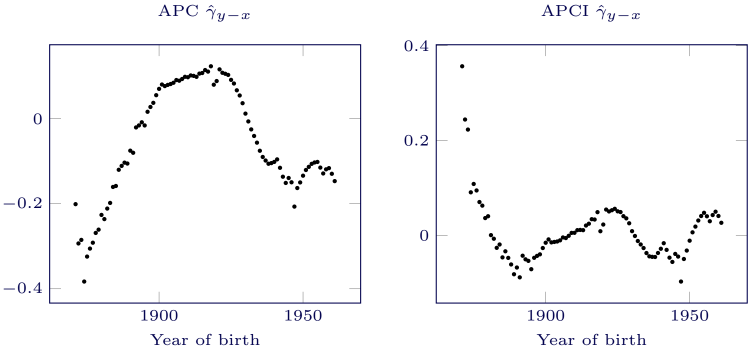


4 κ_y

- κ_y plays a similar role in the Age-Period, APC and Lee-Carter models.
- κ_y plays very different role in the APCI model.
- APCI $\hat{\kappa}_y$ values have less of a clear trend pattern for forecasting.
- APCI $\hat{\kappa}_y$ values are strongly influenced by structural decisions made elsewhere in the model.

4 γ_{y-x}

Parameter estimates $\hat{\gamma}_{y-x}$ for APC and APCI models (both unsmoothed).



4 γ_{y-x}

- The γ_{y-x} values play analogous roles in the APC and APCI models...

  ...yet the values taken and the shapes displayed are very different.

- If values and shapes are so different, what do APCI γ_{y-x} values represent?

  ... and what do these values mean when put into the CMI spreadsheet?

5 Smoothing

5 To smooth or not to smooth?

- Continuous Mortality Investigation (2017) smoothes all parameters.
- However, only α_x and β_x exhibit regular behaviour.
- Does it make sense to smooth κ_y and γ_{y-x}?

5 To smooth or not to smooth?

- CMI's smoothing parameter for κ_y is S_κ.
- Value is set subjectively.
- What is the impact of smoothing κ_y?

5 Impact of smoothing κ_y

life expectancies are [...] very sensitive to the choice made for S_κ, with the impact varying across the age range. At ages above 45, changing S_κ by 1 has a greater impact than changing the long-term rate by 0.5%."

Continuous Mortality Investigation (2016, page 42)

See also https://www.longevitas.co.uk/site/informationmatrix/signalornoise.html

5 Impact of smoothing κ_y

- S_κ has a large impact because κ_y collects features left over from other parts of the model structure.
- Indeed, κ_y collects every non-period effect and applies it without any age modulation.
- If κ_y is a "left-over", should one smooth it at all?

6 Value-at-Risk (VaR)

6 Trend risk v. one-year view?

"Whereas a catastrophe can occur in an instant, longevity risk takes decades to unfold"

The Economist (2012)

6 Trend risk v. one-year view

Solution from Richards et al. (2014):

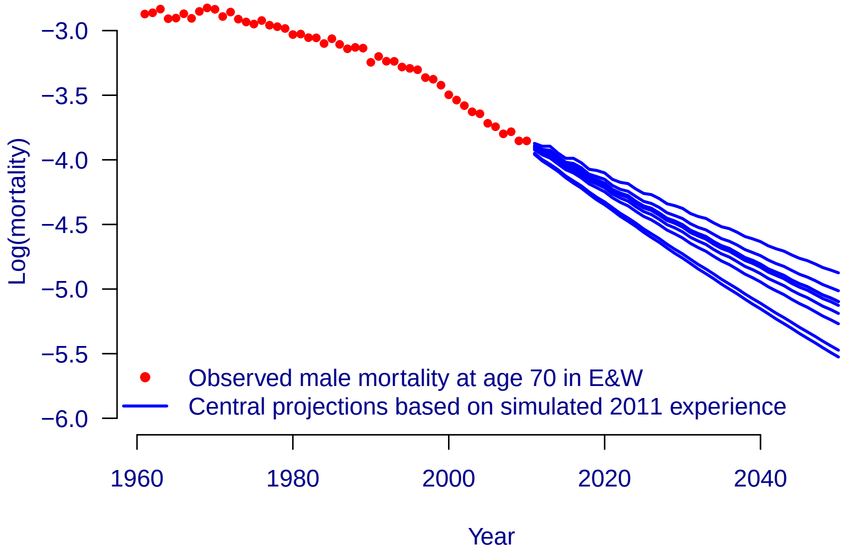
- Simulate next year's experience.
- Refit the model.
- Value liabilities
- Repeat...

6 Forecasting

Approach from Kleinow and Richards (2016) for parameter uncertainty:

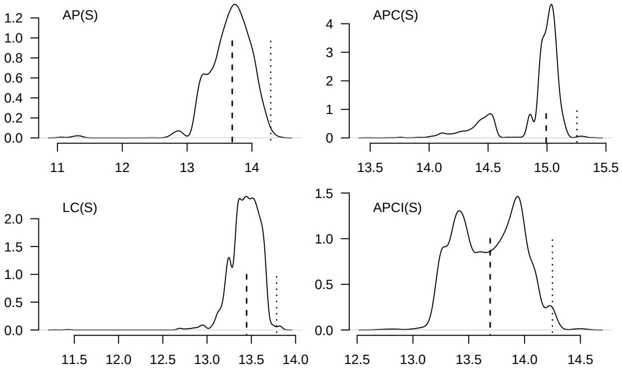
- ARIMA model with mean for κ_y.
- ARIMA model without mean for γ_{y-x}.

6 Sensitivity of forecast



6 Liability densities

Value-at-risk capital requirements for annuities payable to male 70-year-olds. Source: Richards et al. (2017, Table 4).



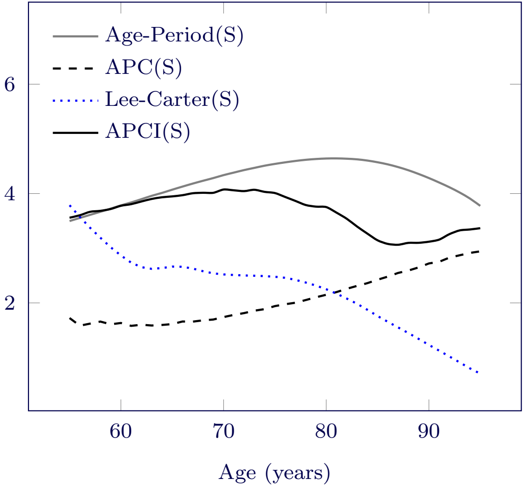
See also https://www.longevitas.co.uk/site/informationmatrix/twinpeaks.html

6 Value-at-risk

- Variety of density shapes.
  $\Rightarrow$ not all unimodal.
- Considerable variability between models.
  $\Rightarrow$ need to use multiple models.

6 Value-at-risk

VaR99.5% capital-requirement percentages by age for four models. Source: Richards et al. (2017).



6 Value-at-risk

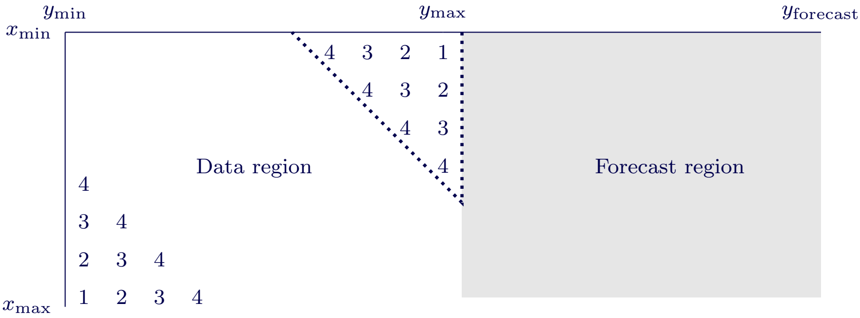
Q. Why do capital requirements reduce with age for Lee-Carter, but not with APCI?

A. κ_y is unmodulated by age in APCI model.

7 Constraints (again)

7 Corner cohorts

Number of observations for each cohort in the data region.



7 Constraints (again)

- Both Continuous Mortality Investigation (2017) and Richards et al. (2017) avoid estimating "corner cohorts".
- This means not all constraints are required for identifiability.
- Continuous Mortality Investigation (2017) and Richards et al. (2017) both fit over-constrained APCI models.
- What impact does this have?

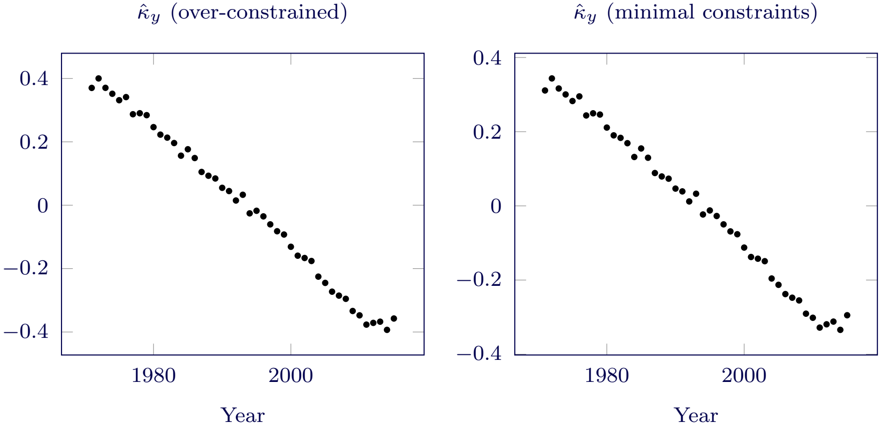
7 Constraints (again)

- Over-constrained models reduce the goodness-of-fit...

  ...but can be used to impose desirable behaviour on parameters.

7 APC model — κ_y

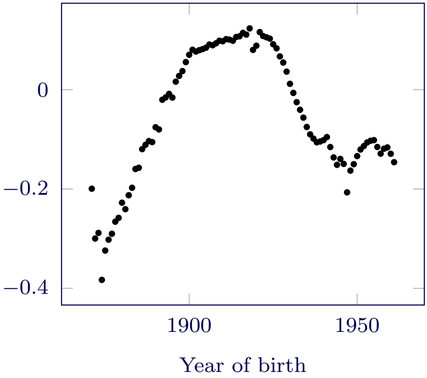
Parameter estimates $\hat{\kappa}_y$ APC(S) model



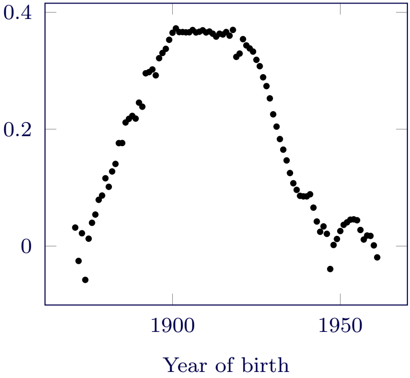
7 APC model — γ_{y-x}

Parameter estimates $\hat{\gamma}_{y-x}$ APC(S) model

$\hat{\gamma}_{y-x}$ (over-constrained)



$\hat{\gamma}_{y-x}$ (minimal constraints)

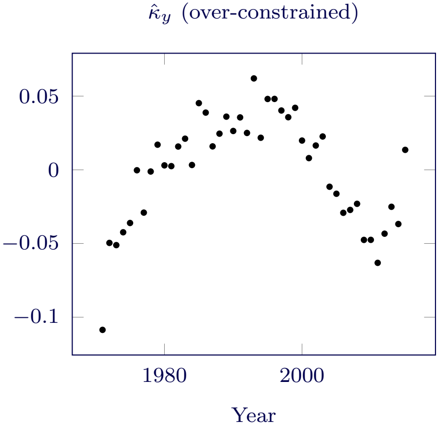


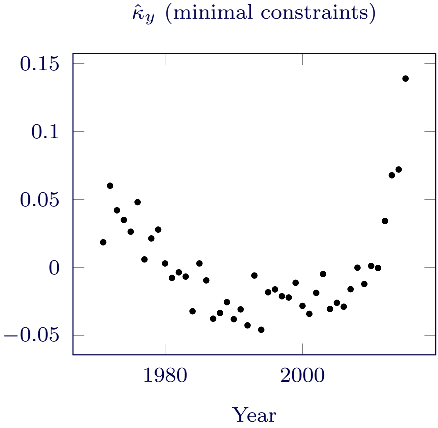
7 APC model

- $\hat{\kappa}_y$ robust to over-constrained model.
- Values for $\hat{\gamma}_{y-x}$ differ, but shape similar.

7 APCI model — κ_y

Parameter estimates $\hat{\kappa}_y$ APCI(S) model

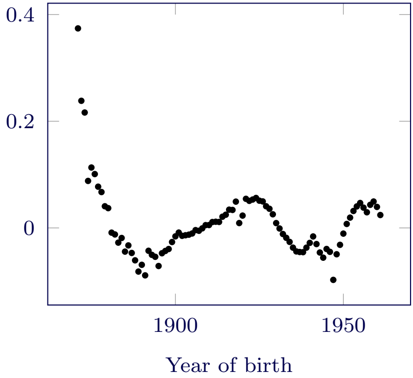




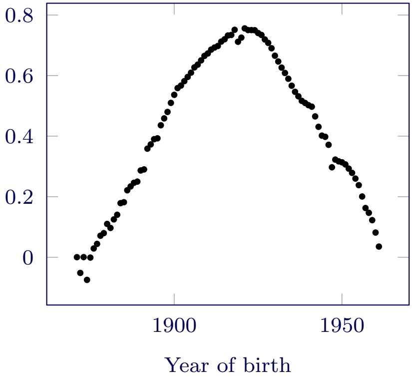
7 APCI model — γ_{y-x}

Parameter estimates $\hat{\gamma}_{y-x}$ APCI(S) model

$\hat{\gamma}_{y-x}$ (over-constrained)



$\hat{\gamma}_{y-x}$ (minimal constraints)



7 APCI model

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- Neither $\hat{\kappa}_y$ nor $\hat{\gamma}_{y-x}$ robust to over-constrained model.
- κ_y in APCI model is a term which picks up left-over aspects of fit.
- $\hat{\gamma}_{y-x}$ changes radically depending on constraint choices.

  $\Rightarrow$ What are the implications for the CMI model of using $\hat{\gamma}_{y-x}$ from APCI model?

8 Conclusions

8 Conclusions

- APCI model is interesting addition to model pantheon.
- APCI model shares features with APC and Lee-Carter models.
- Smoothing $\hat{\alpha}_x$ and $\hat{\beta}_x$ seems sensible.
- Smoothing $\hat{\kappa}_y$ and $\hat{\gamma}_{y-x}$ is not sensible.
- APCI $\hat{\kappa}_y$ and $\hat{\gamma}_{y-x}$ sensitive to constraint choices.

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More on longevity risk at www.longevitas.co.uk