Deutsche Bank, Great Winchester Street, London.

Aspects of longevity basis risk

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23rd April 2012

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1. About the speaker

1. About the speaker

- Consultant on longevity risk since 2005.
- Founded longevity-related analytics businesses in 2006:

- Joint venture with Heriot-Watt in 2009:

2. Background

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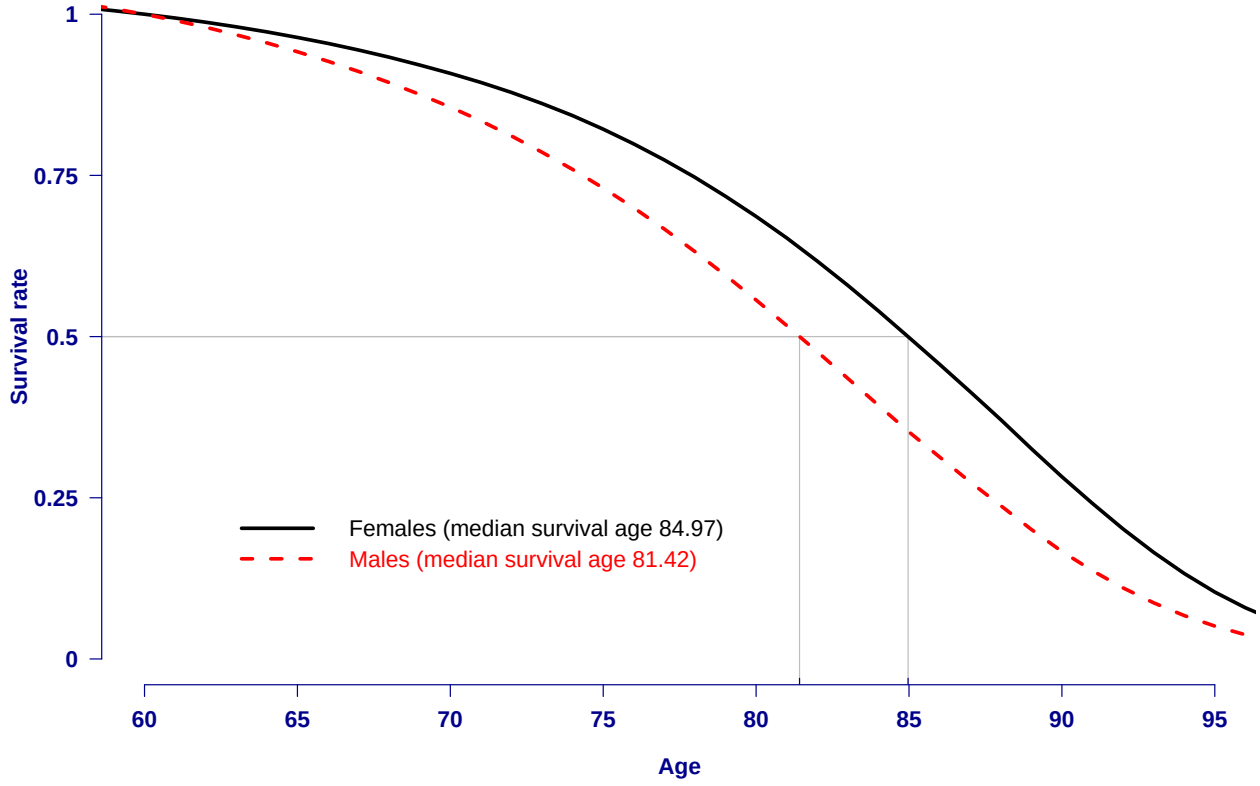
"Longevity risk is the risk that someone lives longer than expected and results in a financial loss"

2. Background

The basic tool for analysing longevity risk is the survival curve:



Source: Own calculations using GAD interim life tables for 2004–2006.

2. Background

"Basis risk is the risk that your longevity liabilities have different characteristics than your reference model"

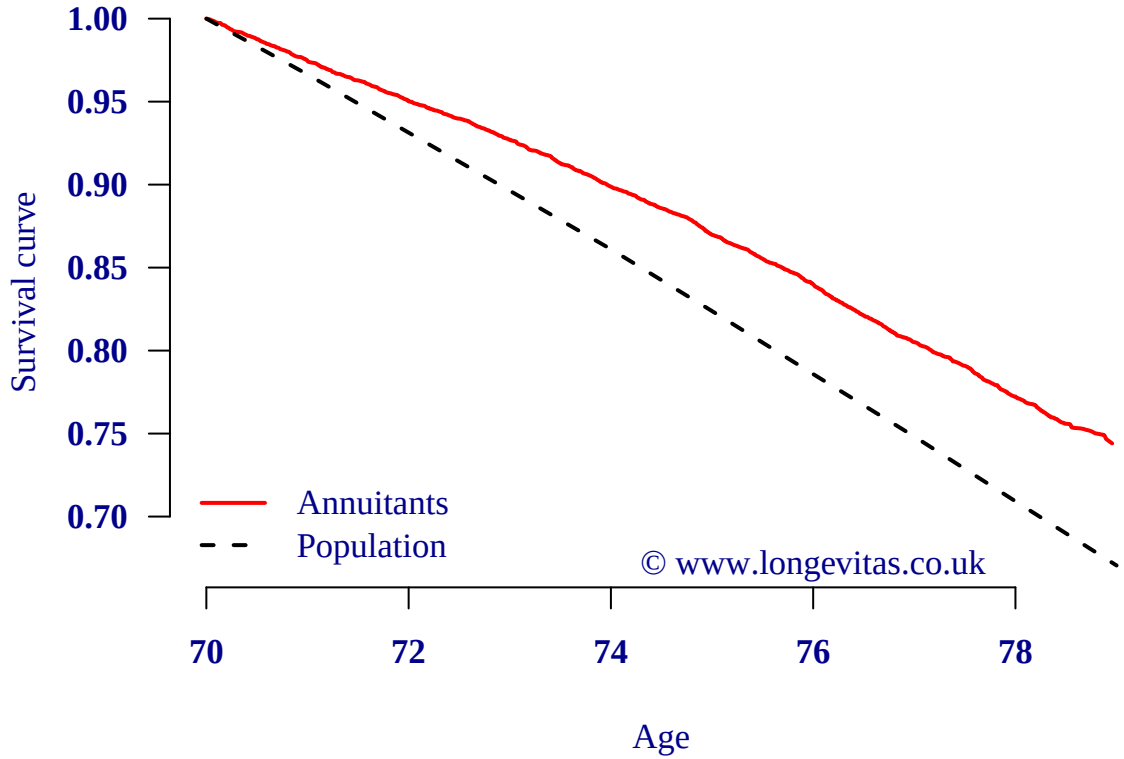
or

"Basis risk is the risk that your hedging instruments do not behave like your liabilities"

3. Basis risk

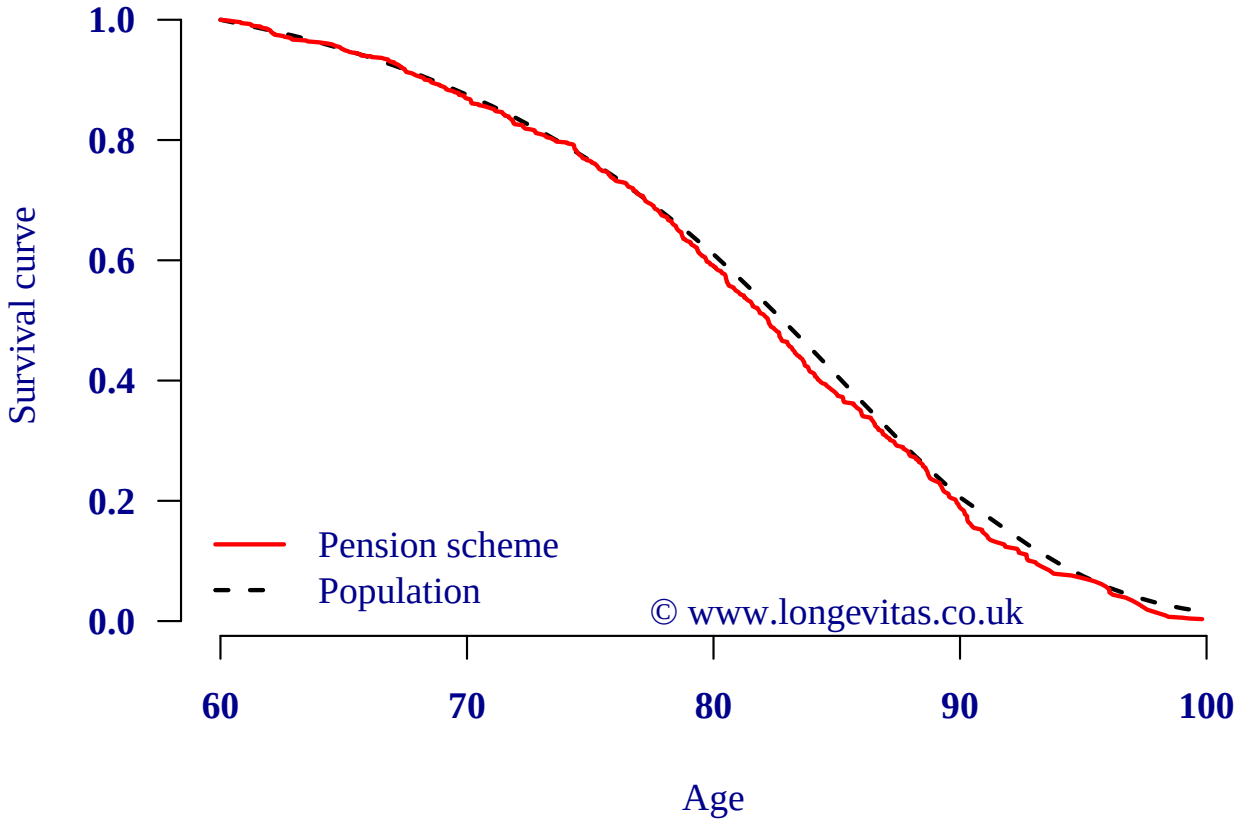
3. Basis risk for annuitants

Survival curve for males born in 1928:



Source: Own calculations using ONS data for males in England and Wales, 1998–2006, and life-office annuitants over the same date range. Kaplan-Meier survival curve for annuitants calculated according to Richards (2010).

3. Basis risk for defined-benefit pensioners



Source: Own calculations using ONS data for males in England and Wales in 2009 and mortality experience of large pension scheme in 2009. Kaplan-Meier survival curve for pension scheme calculated according to Richards (2010).

3. Basis risk for defined-benefit pensioners

- Contemporaneous survival curves are similar in previous slide.
- So where is the basis risk?

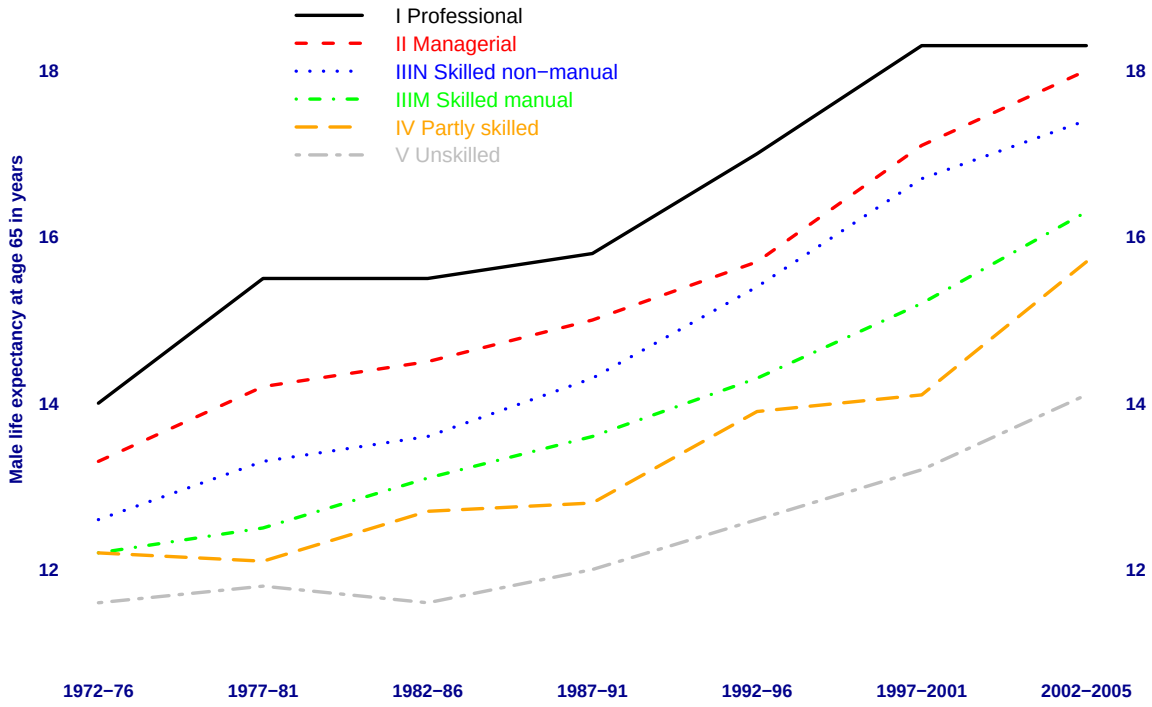
4. Concentration risk

4. Concentration risk

- Different sub-groups can have different mortality rates.
- Sub-groups can have very different impacts on liabilities...

4. Concentration risk

Period life expectancy for males aged 65 by socio-economic group:



Source: ONS Longitudinal Survey.

4. Concentration risk

Scheme liabilities by pension quintile:

Pension quintile	Number of lives	Average pension p.a.	Proportion of scheme pension p.a.
1	2,985	£873	3%
2	2,985	£2,572	9%
3	2,985	£4,453	15%
4	2,985	£7,075	24%
5	2,985	£14,835	50%

- Half of pensions are paid to just 20% of membership!
- Annuity portfolios typically even more concentrated — see Richards (2008).

Source: Own calculations using data for large pension scheme in England and Wales.

4. Concentration risk

Complete period life expectancies at age 65 by pension quintile:

Pension quintile	Life expectancy: (a) males	(b) females
1–4	17.2	20.4
5	18.4	21.7

Source: Own calculations using mortality-experience data for large pension scheme in England and Wales in 2009.

4. Concentration risk

- Basis risk can arise due to concentration risk.
- Many more risk factors to consider...

4. Concentration risk

Financial impact of mortality rating factors:

Factor	Step change	Reserve	Change
Base case	-	13.39	-
Gender	Female-male	12.14	-9.3%
Lifestyle	Top-bottom	10.94	-9.9%
Duration	Short-long	9.88	-9.7%
Pension size	Large-small	9.36	-5.2%
Region	South-North	8.90	-4.9%
Overall	-	-	-33.6%

Source: Richards and Jones (2004), page 39.

5. Hedging

5. Hedging

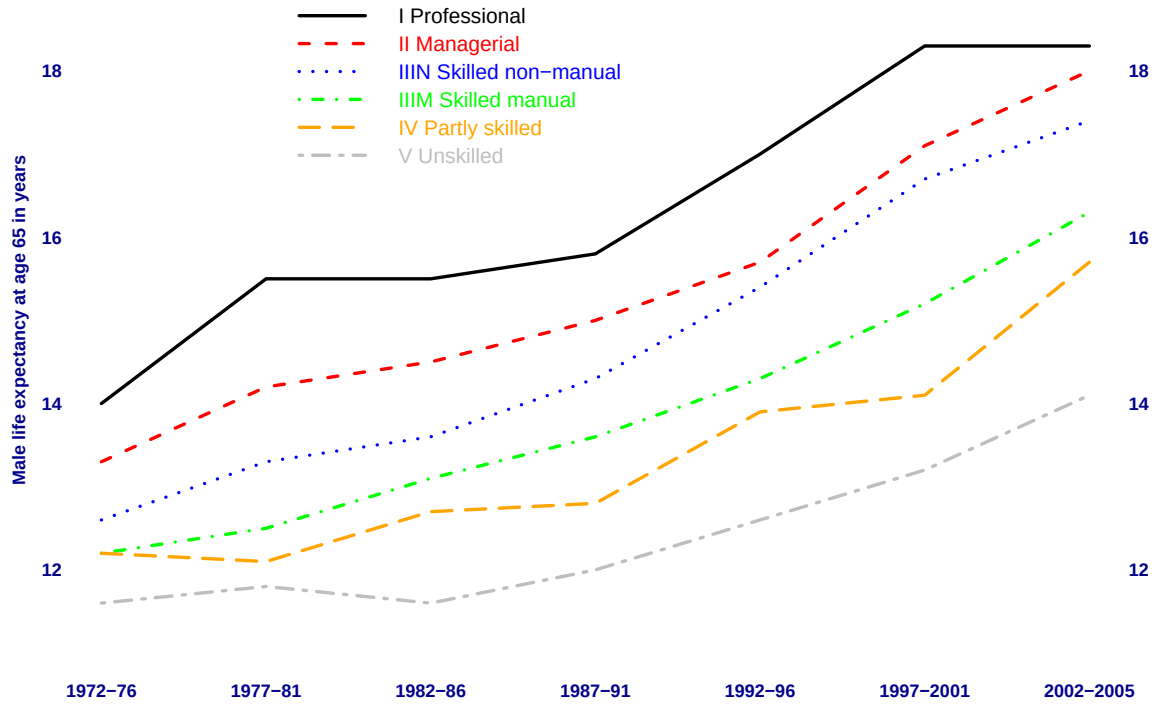
- Numerous derivatives exist for hedging longevity risk.
- A *survivor forward* is a type of derivative based on the survival curve:

$$\text{Payoff} = \text{Nominal} \times (\text{actual survival rate} - \text{strike rate})$$

- Survival rate based on population data, so basis risk exists.

5. Hedging

Note correlations between trends by socio-economic group:



Source: ONS Longitudinal Survey.

5. Hedging

- *Some* protection must be afforded by population-based derivatives.
- Key questions:
    1. Which hedging assets best suit your portfolio?
    2. How much protection do you get from the hedging assets?
    3. How much is that protection worth?

5. Hedging

The *hedge effectiveness* is:

$$\left(1 - \frac{\text{capital requirement with hedging assets}}{\text{capital requirement without hedging assets}}\right) \times 100\%$$

- Ideal is 100%, i.e. capital requirement reduced to zero.
- A useless hedge would have 0%, i.e. capital requirement unchanged.
- A worse-than-useless hedge would have a negative effectiveness, i.e. capital requirement increased.
- Other definitions of hedge effectiveness exist, e.g. Cairns (2011).

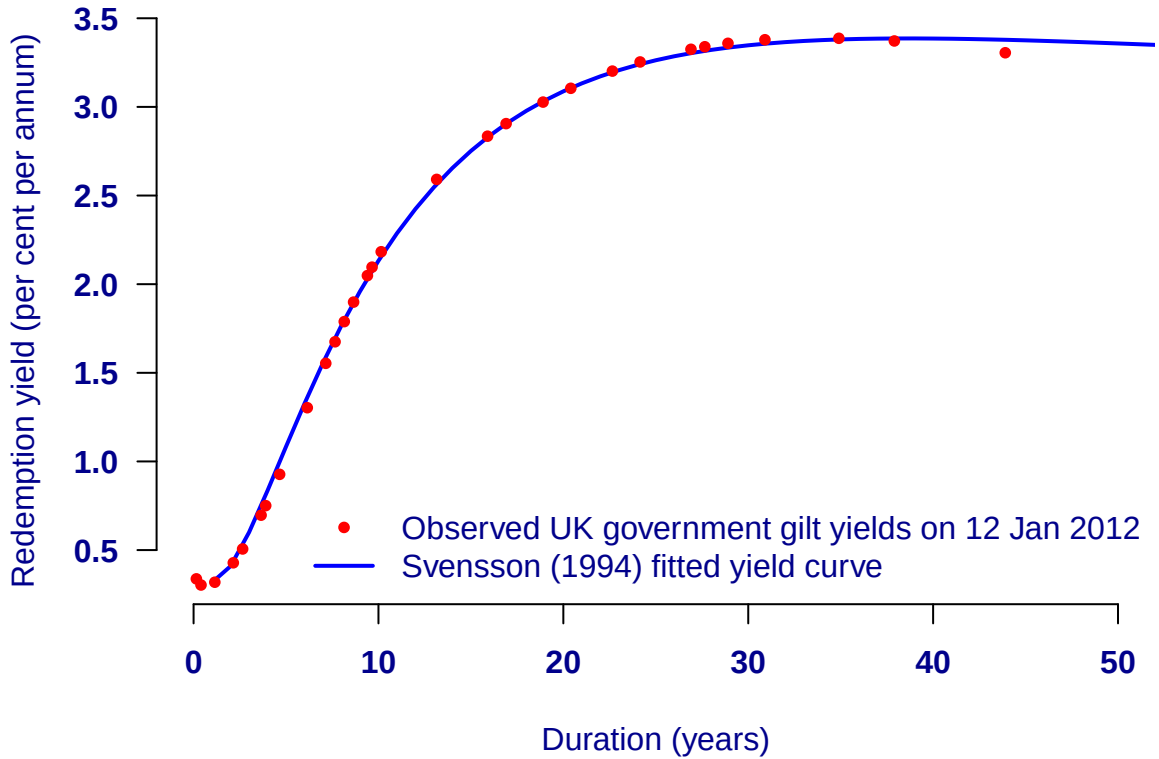
5. Hedging

- Capital requirement is 99.5th percentile run-off cost.
- Run-off simulations cover all forms of longevity risk.
- Capital therefore covers:
  - (i) trend risk
  - (ii) volatility of annual mortality rates
  - (iii) idiosyncratic risk, including concentration risk

- Survivor forward cannot cover (iii), so 100% effectiveness not expected.

5. Hedging

All cashflows and payoffs discounted using the following yield curve:



Source: Observed redemption yields implied by prices of principal strips of UK gilts on 12th January 2012 (●) and fitted Svensson (1994) model (——).

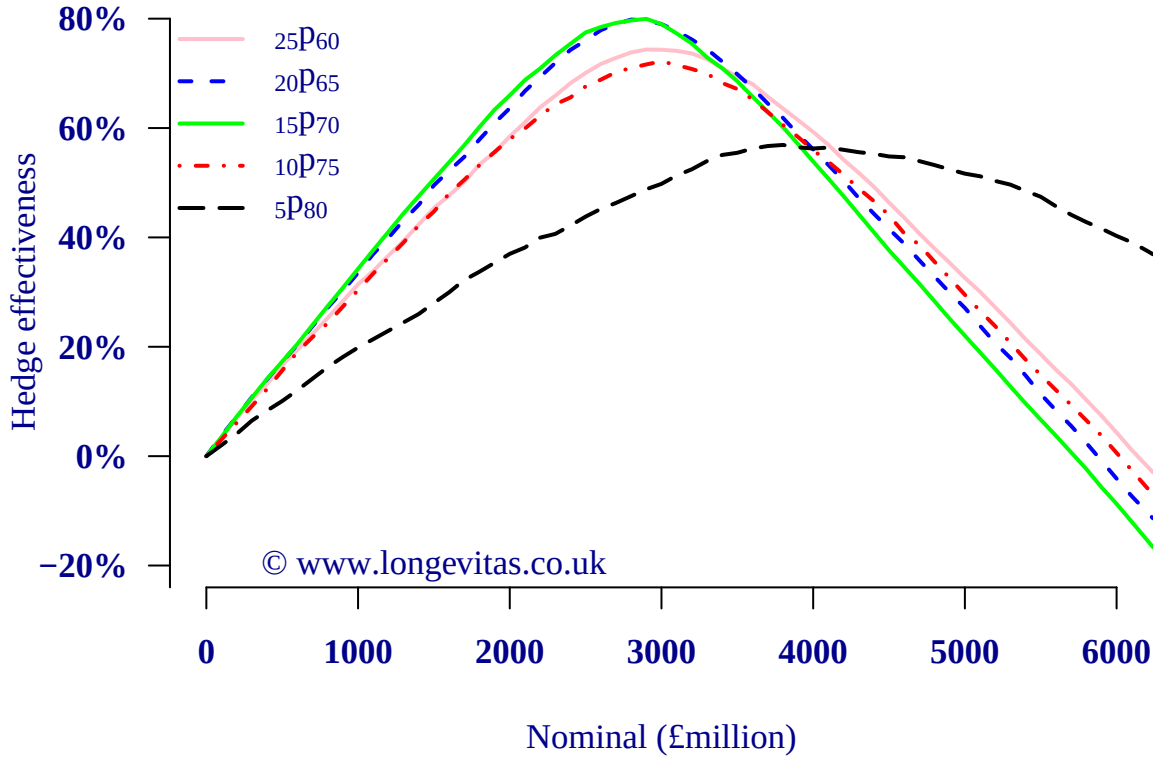
5. Hedging options

- Buy-out or Part VII transfer 100% effective, but takes time to execute.
- Reinsurance could be 100% effective, but introduces counterparty risk.
- What can index-based derivatives offer?

5. Hedge effectiveness for large pension scheme

- Consider a large pension scheme in England.
- Run-off simulations suggest maximum hedge effectiveness around 85%.
- Consider a single survivor forward, ${}_tp_x$, based on population mortality:

5. Hedge effectiveness for large pension scheme



Source: Own calculations based on 10,000 run-off simulations, similar in manner to those in Richards and Currie (2009). Hedging effectiveness for various survivor forwards defined on male mortality for England and Wales. Mortality forecasts according to Lee-Carter (1992) model with smoothed coefficients by age. Survivor forwards commenced in 2011.

6. Conclusions

- Basis risk is real, but can be measured.
- Detailed analysis of risk factors in a portfolio is essential.
- Population-based hedges do provide some protection...
- ...the only question is how much?

References — I

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