IFoA sessional meeting

Allowing for shocks in portfolio mortality models

Stephen J. Richards

Wednesday 15th December 2021, 10:00hrs

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Motivation

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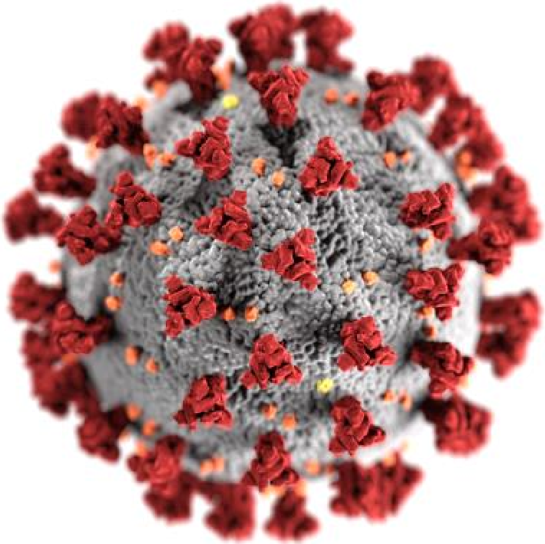

Motivation

- How does COVID-19 affect portfolio mortality?
- How can actuaries avoid bias when deriving bases?

Bias is a business problem

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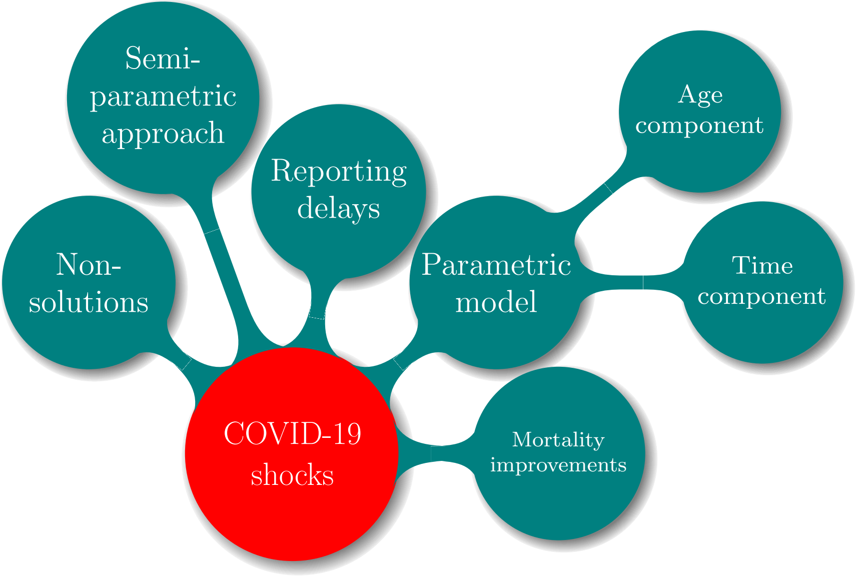
Consider an annuity portfolio or pension scheme:

Reserving

✗ Imprudent to include excess shock mortality when deriving a long-term basis.

Pricing

✗ Including excess shock mortality underprices risk in bulk annuities and longevity swaps.


Semi-parametric approach
Reporting delays
Age component
Non-solutions
Parametric model
Time component
COVID-19 shocks
Mortality improvements

All-cause deaths aged 65+

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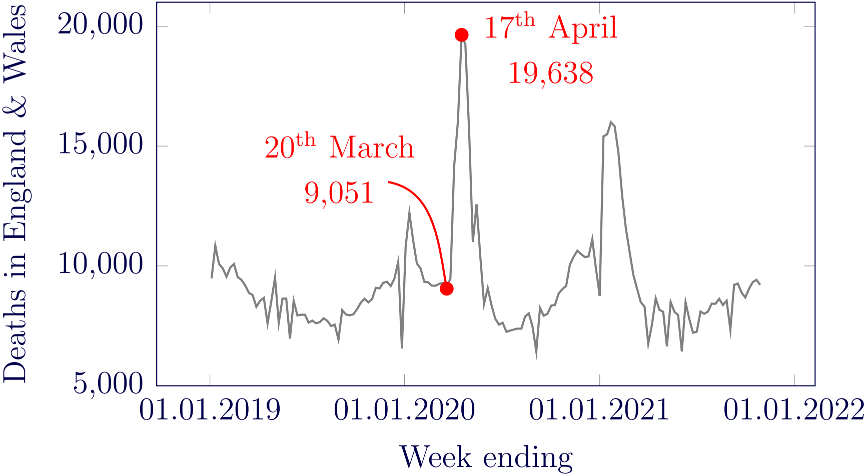

Source: ONS data for deaths in England & Wales by date of registration.

COVID-19 deaths

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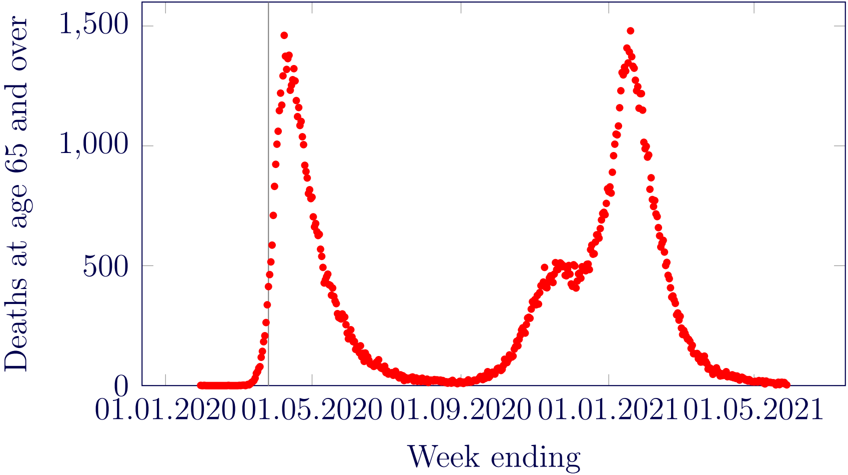
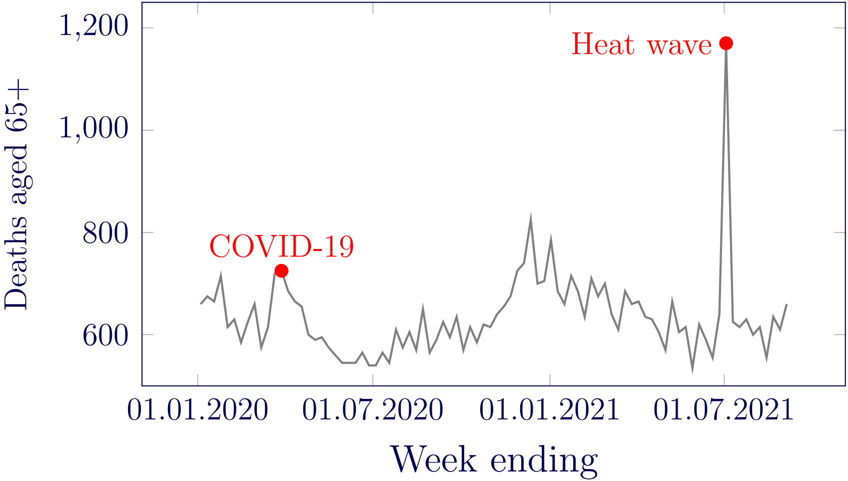
UK deaths where COVID-19 was listed as one of the causes.



Source: ONS data.

COVID-19 v. climate stress

Deaths in British Columbia aged 65+



Source: Statistics Canada.

Mortality shocks

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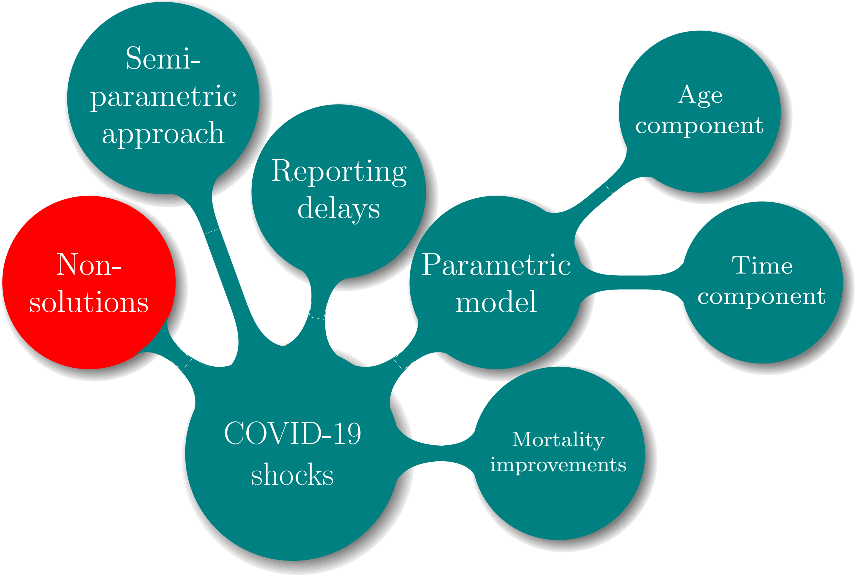
- Shocks occur over a very short space of time.
- Annual rates cannot capture the nature of shocks.
- Continuous-time methods are required.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Build a cause-of-death model?

- ✗ Pension schemes don't record cause of death.
- ✗ Cause of death not wholly reliable anyway.

Ignore periods including shocks?

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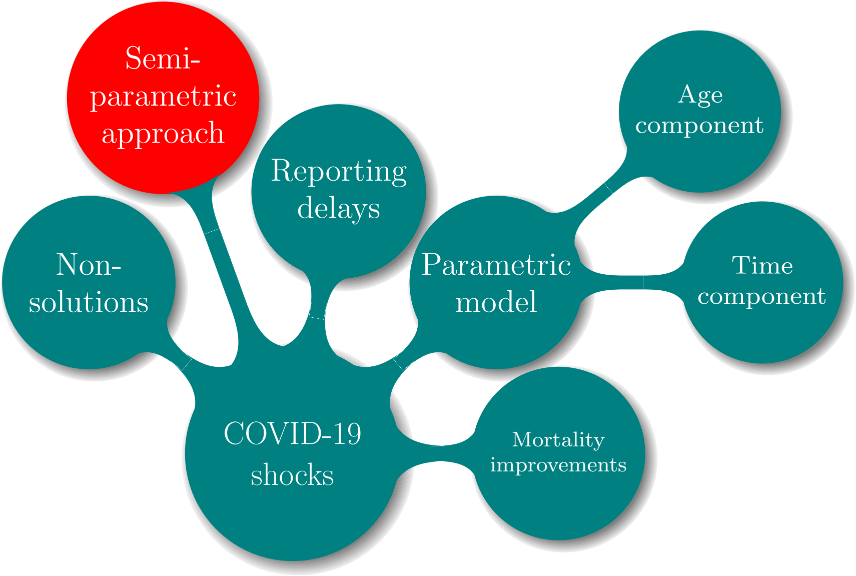
- ✗ Often only have data for last 3–5 years.
- ✗ If your model can't handle the data, you need a better model!

Requirements

Need methodologies that:

- Work with the data actually available,
- Work with the whole exposure period, and
- Can handle sharp spikes in mortality.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Definitions

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y	Start of period of interest.
$y + t_i$	Unique date with at least one death.
d_{y+t_i}	Number of deaths at $y + t_i$.
$l_{y+t_i^-}$	Number of lives immediately before $y + t_i$.

Nelson-Aalen estimator

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$$\hat{\Lambda}_{y,t} = \sum_{t_i \le t} \frac{d_{y+t_i}}{l_{y+t_i^-}} \qquad (1)$$

$\hat{\Lambda}_{y,t}$ estimates the integrated hazard over time.

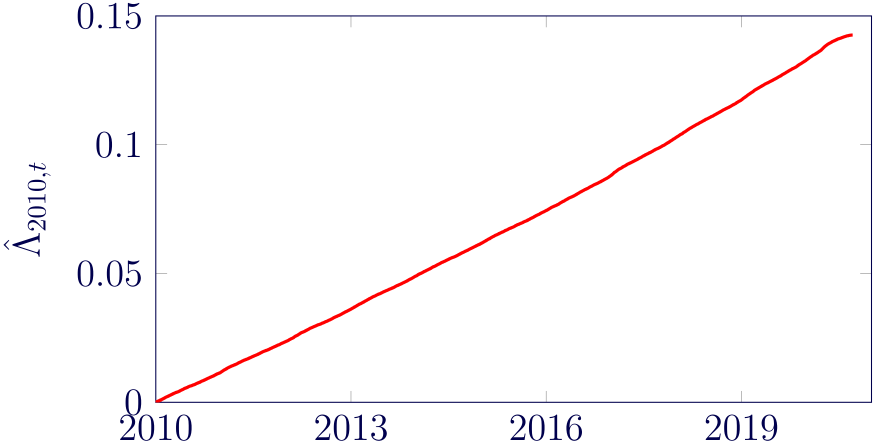
Source: Richards [2021a, equation 1].

Three international portfolios

FRA	AVC-like top-up annuities.
UK3	DC annuities.
USA3	DB buy-out annuities.

Source: Richards [2021a, Table 1]. Data extracts taken in September 2020.

FRA portfolio, $\hat{\Lambda}_{2010,t}$



Source: Richards [2021a, Figure 2(a)].

Semi-parametric estimator

First central difference around $\hat{\Lambda}_{y,t}$:

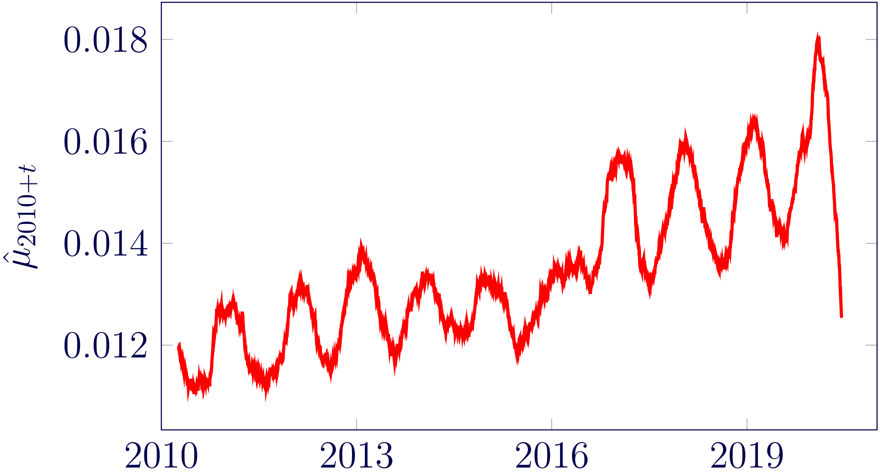
$$\hat{\mu}_{y+t} = \frac{\hat{\Lambda}_{y,t+\frac{c}{2}} - \hat{\Lambda}_{y,t-\frac{c}{2}}}{c} \tag{2}$$

where $c > 0$ is the bandwidth parameter.

$\hat{\mu}_{y+t}$ is the portfolio hazard estimate in time.

Source: Richards [2021a, equation 2].

FRA, $\hat{\mu}_{2010+t}$, $c = 0.5$



Source: Richards [2021a, Figure 2(b)].

Non-parametric estimator

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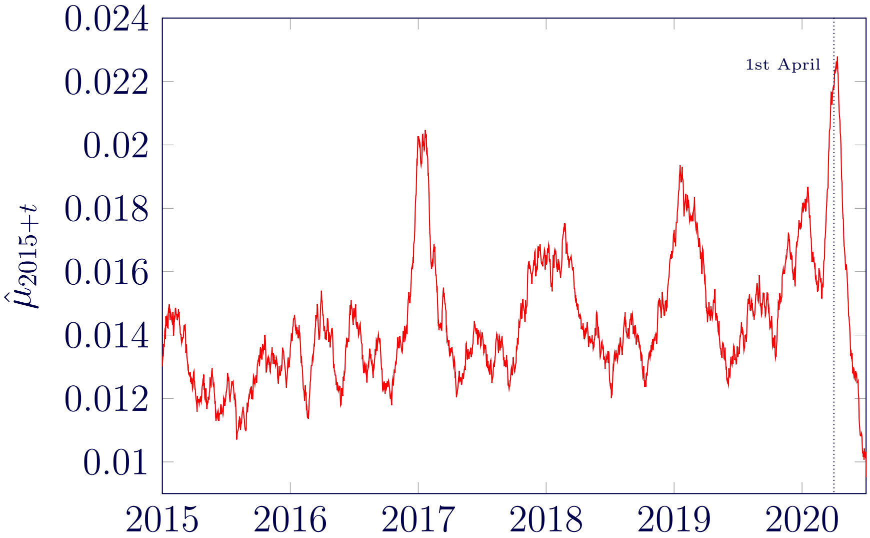
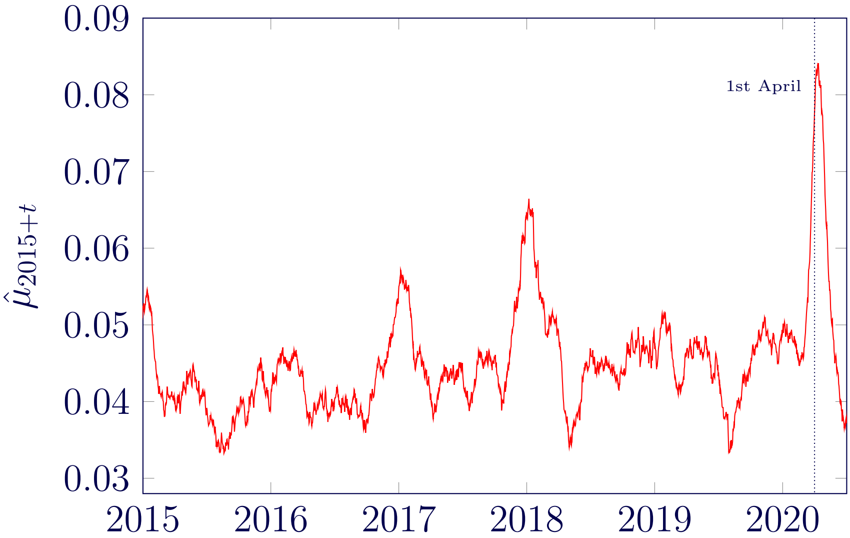
- $\hat{\Lambda}_{y,t}$ might appear rather featureless...
  ...but $\hat{\mu}_{y+t}$ reveals rich detail of seasonal patterns.
- Reducing c reveals the first COVID-19 shock...

FRA $\hat{\mu}_{2015+t}$, $c = 0.2$



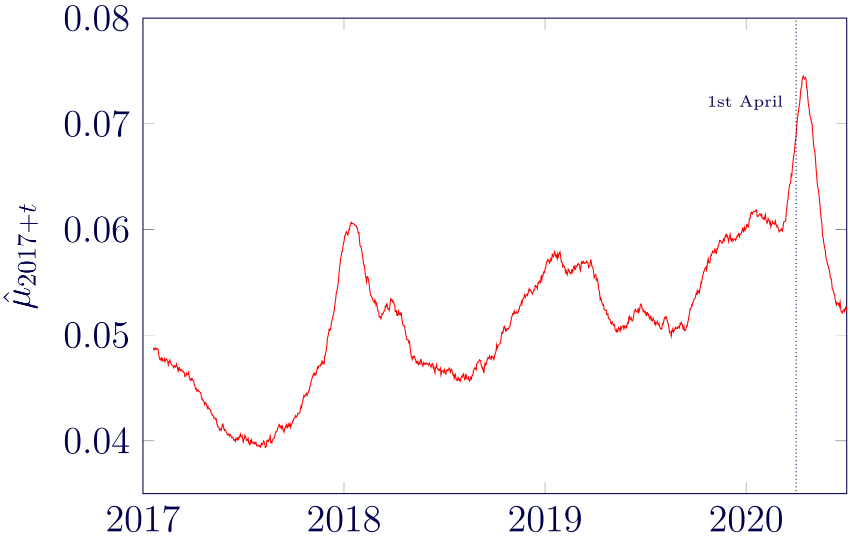
Source: Richards [2021a, Figure 3(a)].

UK3 $\hat{\mu}_{2015+t}$, $c = 0.2$



Source: Richards [2021a, Figure 3(b)].

USA3 $\hat{\mu}_{2017+t}$, $c = 0.2$



Source: Richards [2021a, Figure 3(c)].

Semi-parametric estimator

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First COVID-19 shock hit French, UK and US annuity portfolios at the same time, peaking in April 2020.

Data privacy

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- $\hat{\Lambda}_{y,t}$ and $\hat{\mu}_{y+t}$ need only:
  - Date of annuity commencement,
  - Date of cessation, and
  - Nature of cessation (death, withdrawal etc).
- No personal data required.
- GDPR does not apply.

Semi-parametric estimator

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Advantages:

- Reveals seasonal variation.
- Reveals mortality shocks.
- Requires no personal data (GDPR-safe).
- Easy to implement in spreadsheet or R.

Semi-parametric estimator

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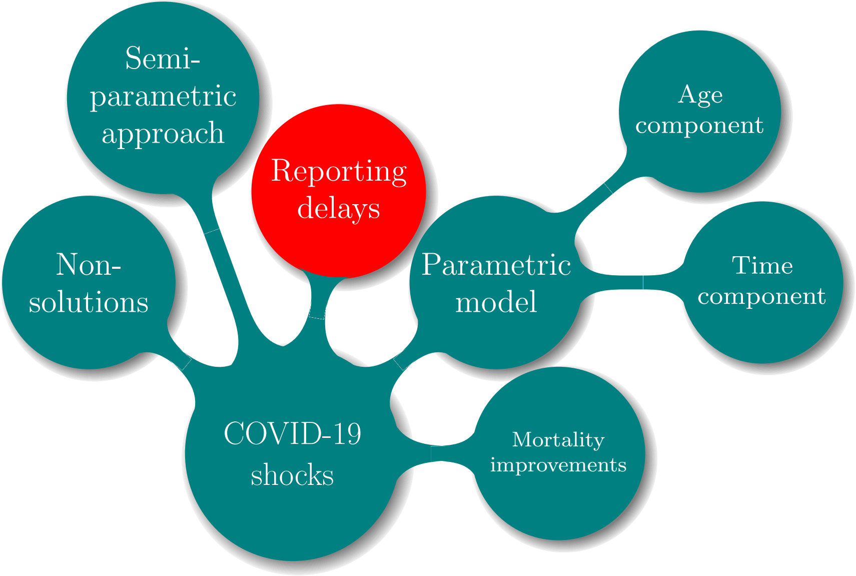
Drawbacks:

- Ignores key risk factors like age.
- Undefined for most recent $\frac{c}{2}$ years.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Reporting delays

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- Delays in reporting deaths to administrator.
- Consider two extracts at times $u_1 < u_2$.
- Calculate ratio, R, of $\hat{\mu}_{y+t}$ using each extract.

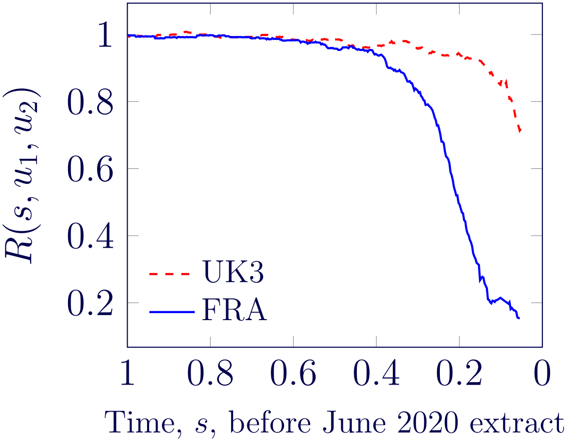
Reporting delays

$$R(s, u_1, u_2) = \frac{\hat{\mu}_{u_1-s} \text{ using extract at time } u_1}{\hat{\mu}_{u_1-s} \text{ using extract at time } u_2} \qquad (3)$$

Source: Richards [2021a, equation (3)].

Reporting delays

Proportion of deaths reported by June 2020:



Source: Richards [2021a, Figure 7].

Reporting delays

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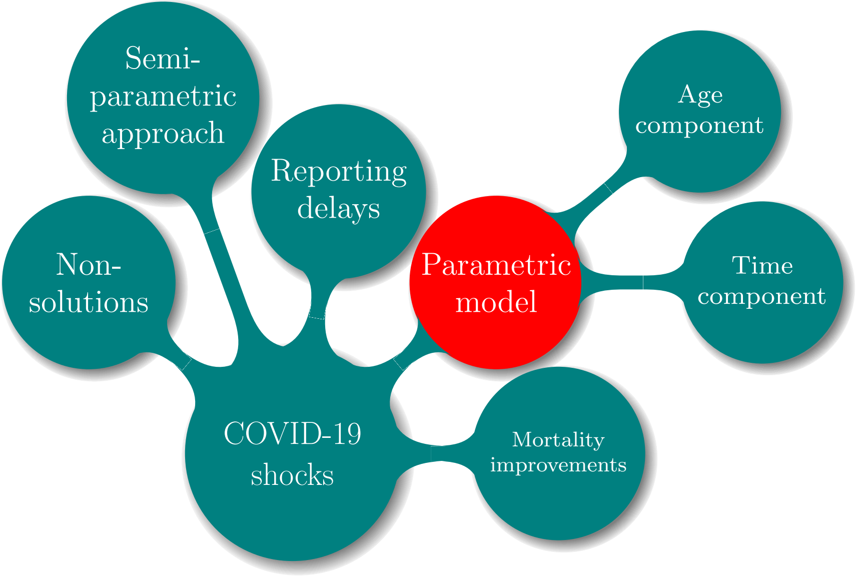
- Reporting delays are another source of bias.
- Could ignore exposure period prior to extract.
- Better to not discard data (see later solution).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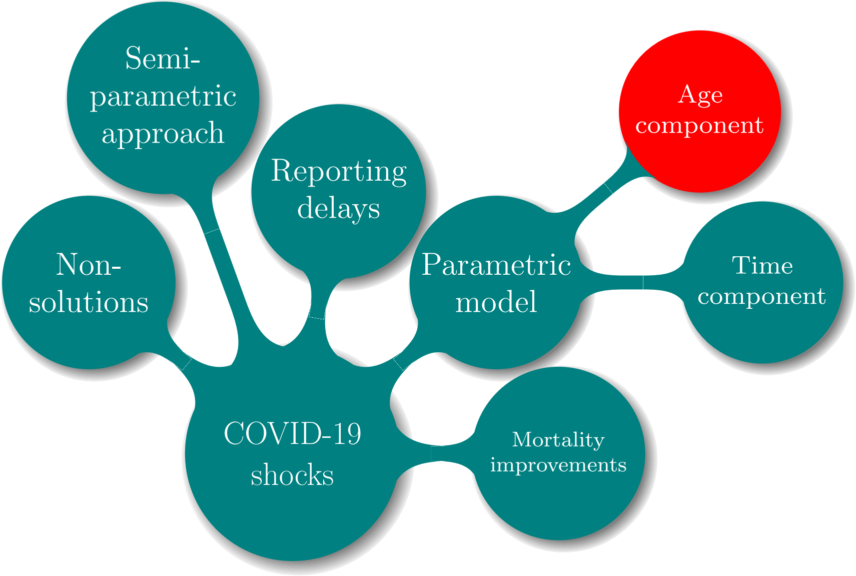
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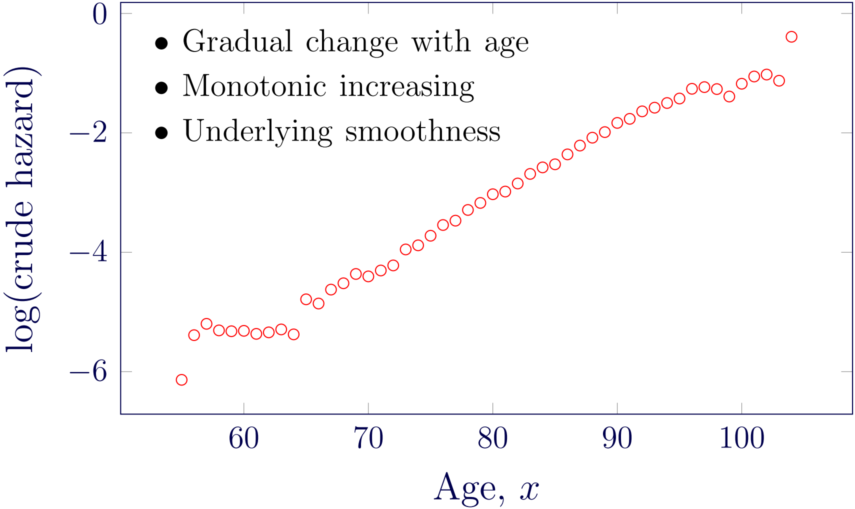
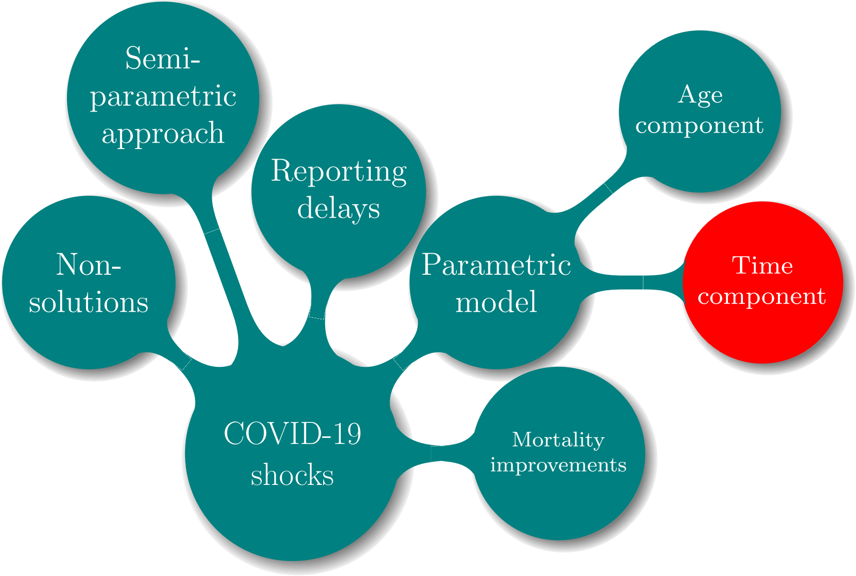
Mortality by age



Source: Richards [2021b, Figure 5(a)]. UK3 data set, 2015–2019.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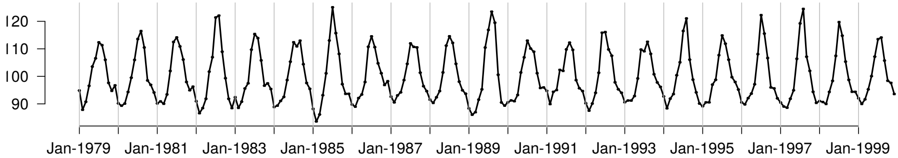
Mortality across time: seasons

Percentage of average daily number of deaths in Australia, all causes, 1979–1999.



Source: de Looper [2002].

Even without a pandemic, mortality levels fluctuate in time.

Mortality in time

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- Not monotonic (ever).
- But smooth on a day-to-day basis, even during a pandemic. . .

COVID-19 deaths

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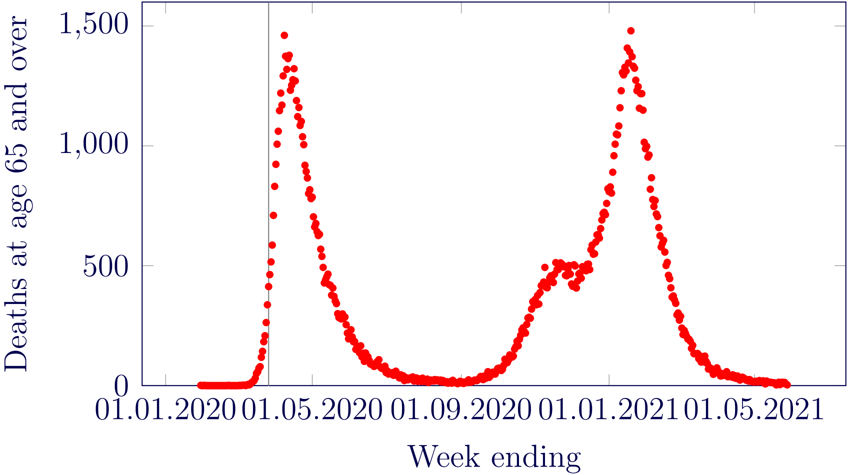
UK deaths where COVID-19 was listed as one of the causes.



Source: ONS data.

Modelling requirements

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Mortality by age

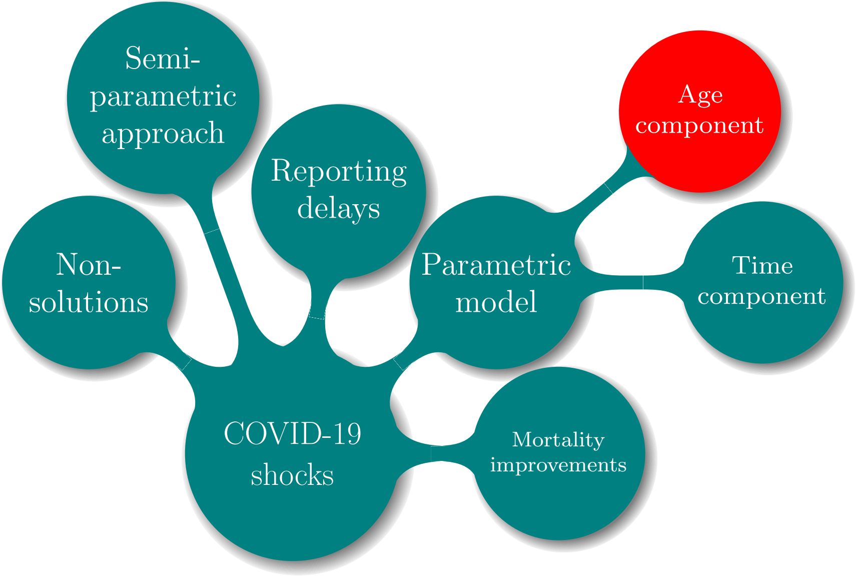
- Slow, monotonic changes.
- Little flexibility needed.

Mortality by time

- Fast, non-monotonic changes.
- Greater flexibility needed.

Continuous age-period model

$$\log \mu_{x,y} = \underset{\text{age component}}{\text{Monotonic}} + \underset{\text{period component}}{\text{Locally flexible}}$$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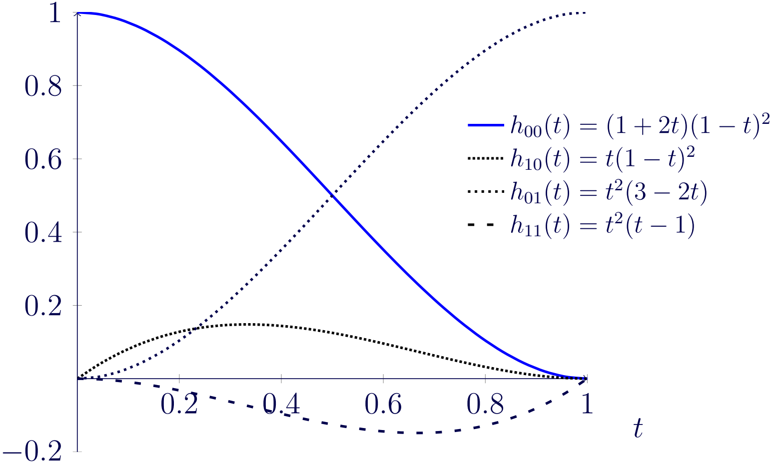
A basis of Hermite splines



Source: Richards [2020].

Hermite-spline mortality model

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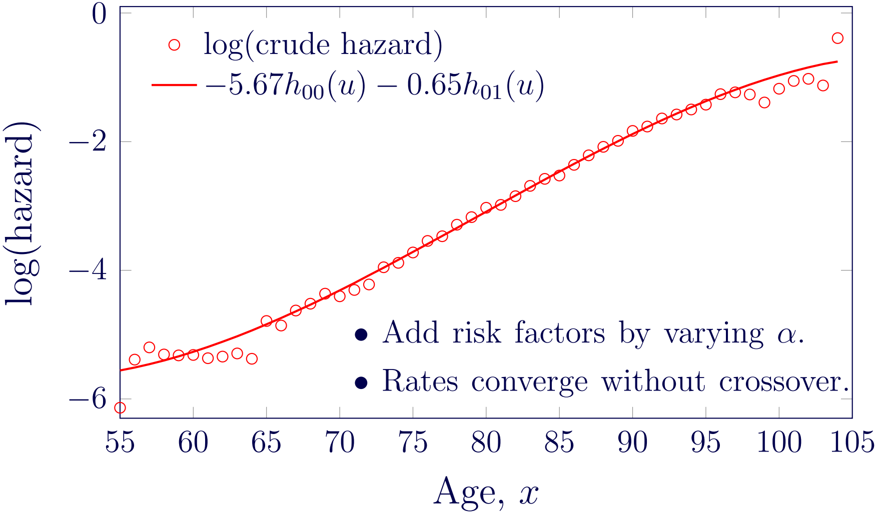
- Age x lies in interval $[x_{\min}, x_{\max}]$.
- Define $u = \dfrac{(x - x_{\min})}{(x_{\max} - x_{\min})}$, so $u \in [0, 1]$.
- $\log \mu_x = \alpha h_{00}(u) + \omega h_{01}(u)$

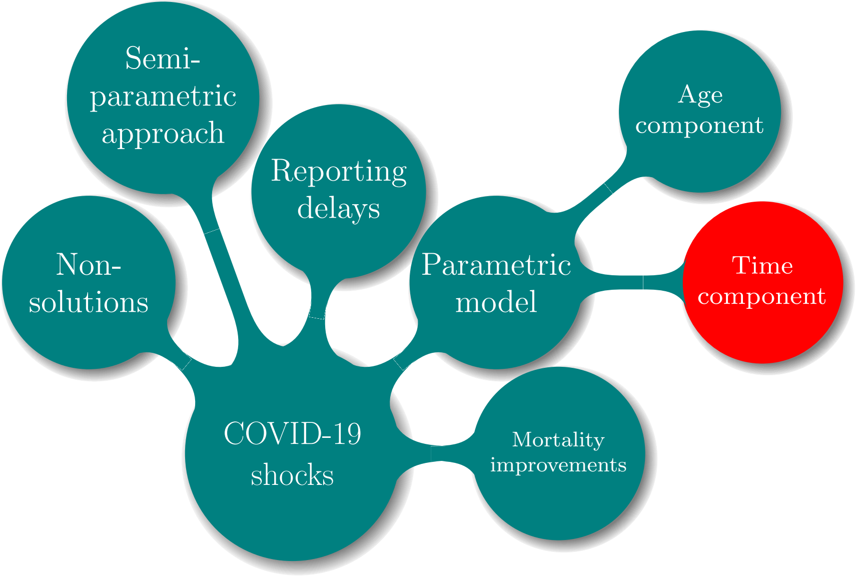
Parameters α and ω can be estimated from data.

Source: Richards [2020].

Hermite-spline mortality model



- Add risk factors by varying α.
- Rates converge without crossover.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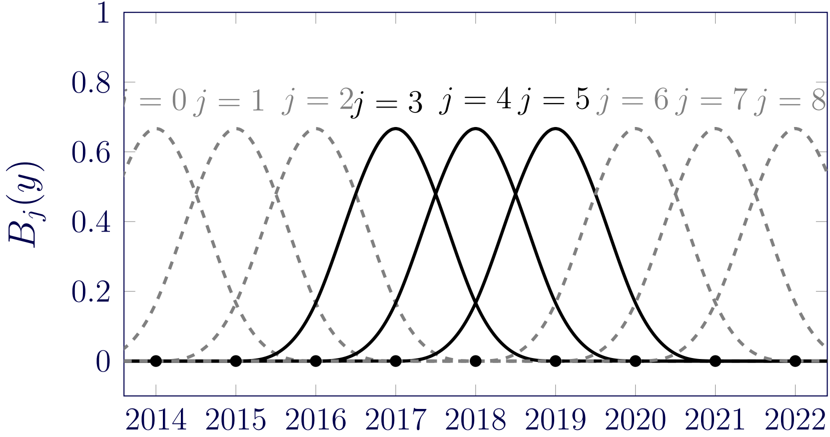
A basis of cubic B-splines

A basis of nine equally-spaced cubic B-splines spanning 1st January 2015 to end-2020, indexed $j = 0, 1, \ldots, 8$.



B-splines

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Define:

- $B_j(y)$ as the j^{th} basis spline at time y.
- $\kappa_{0,j}$, the coefficient of spline B_j.
- $\mu_{x,y}$, the mortality hazard at age x and time y.
- μ_x, the Hermite-spline model for mortality by age.

Continuous age-period model

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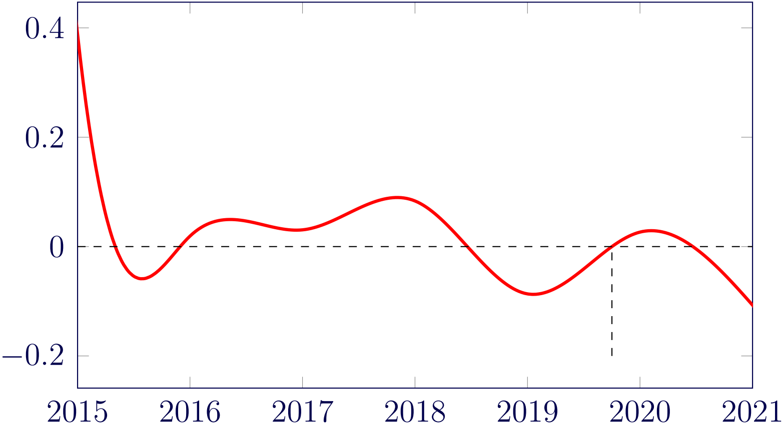
$$\log \mu_{x,y} = \underbrace{\log \mu_x}_{\text{Hermite age component}} + \underbrace{\sum_{j \geq 1} \kappa_{0,j} B_j(y)}_{\text{Schoenberg time component}}$$

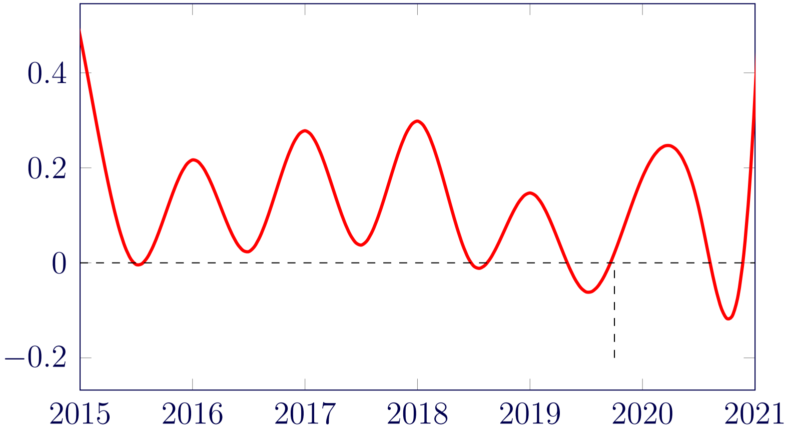
UK3, one knot per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



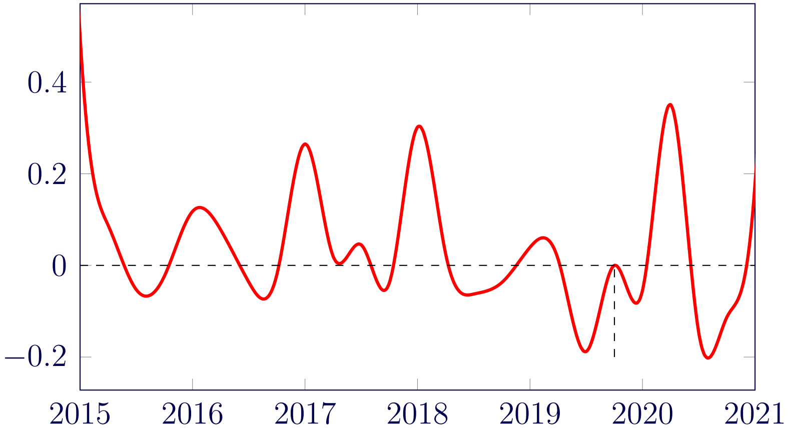
UK3, two knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



UK3, four knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



UK3, ten knots per year

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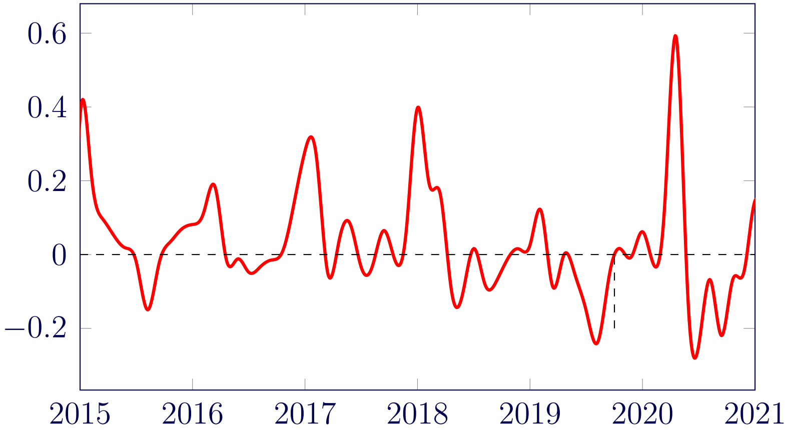
$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



Variable knot spacing

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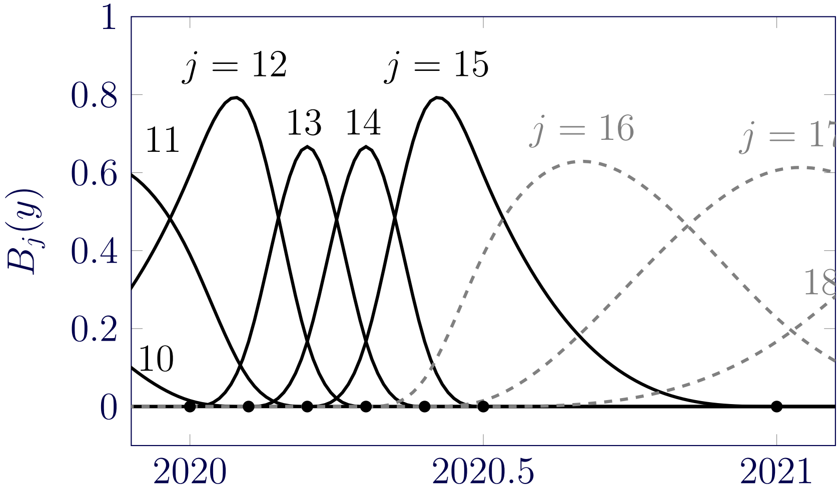
- Knots don't have to be equally spaced [Kaishev et al., 2016].
- Use two knots per year for seasonal variation...

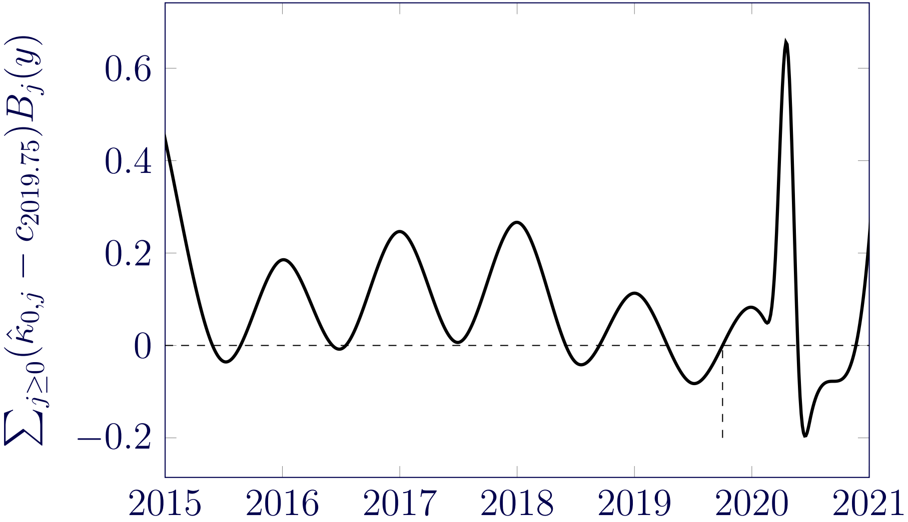
  ...and add knots where we know the shocks are.

Variable knot spacing

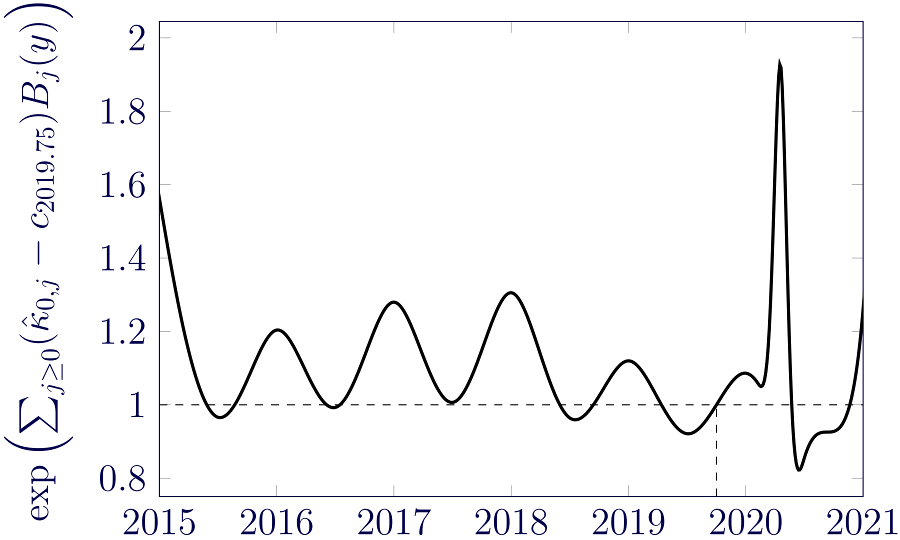
Part of a basis of nineteen variably-spaced cubic B-splines.



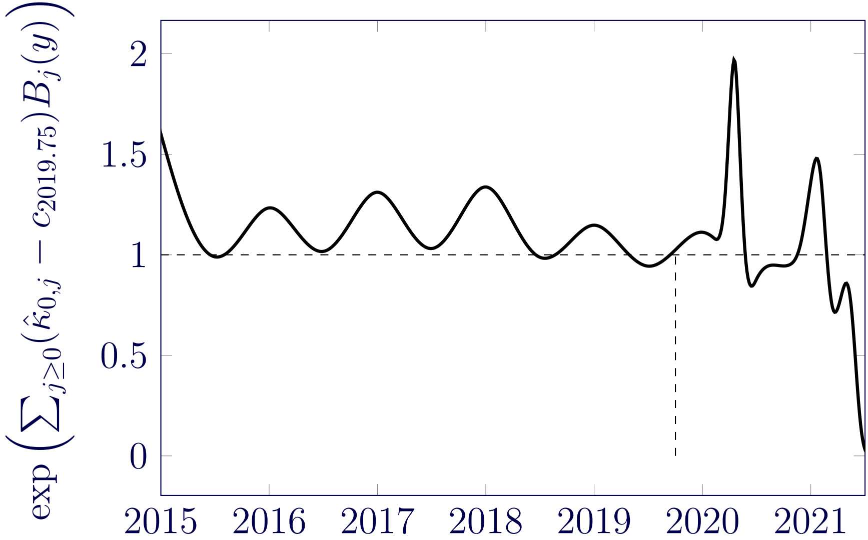
Variable knot spacing

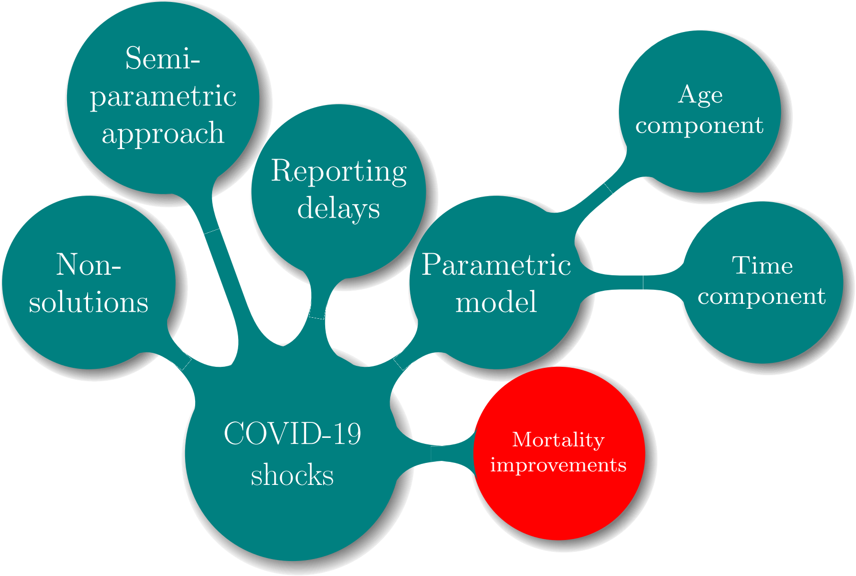


Variable knot spacing



Including shocks and delays

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Mortality improvements

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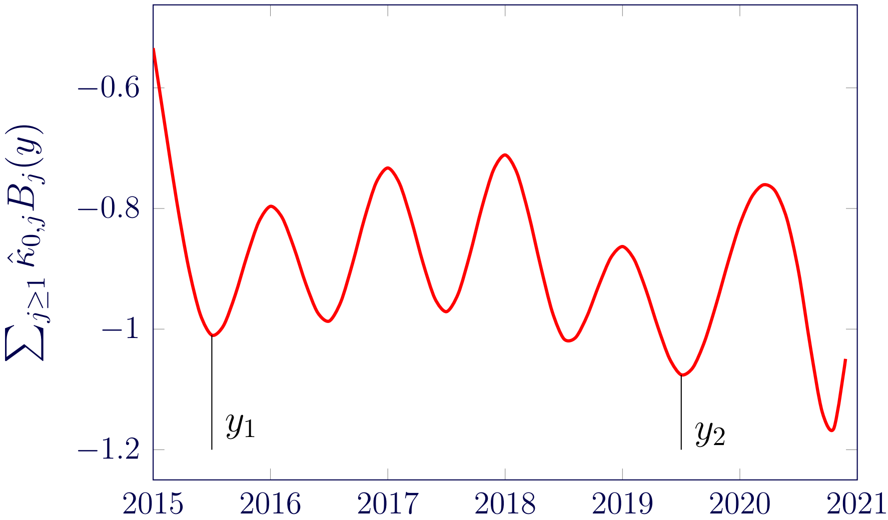
- We can also estimate portfolio-specific mortality improvements.
- Consider $\sum_j \hat{\kappa}_{0,j} B_j(y)$ at two times y_1 and y_2.
- Use midsummer points for y_1 and y_2 for stability.

UK3, two knots per year



Mortality improvement

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Annual improvement rate, i, between y_1 and y_2:

$$i_{y_1,y_2} = \left[1 - \exp\left(\frac{\sum_{j\geq 1} \hat{\kappa}_{0,j}\left[B_j(y_2) - B_j(y_1)\right]}{y_2 - y_1}\right)\right] \times 100\%$$

Source: Richards [2021b, equation 10].

Mortality improvement

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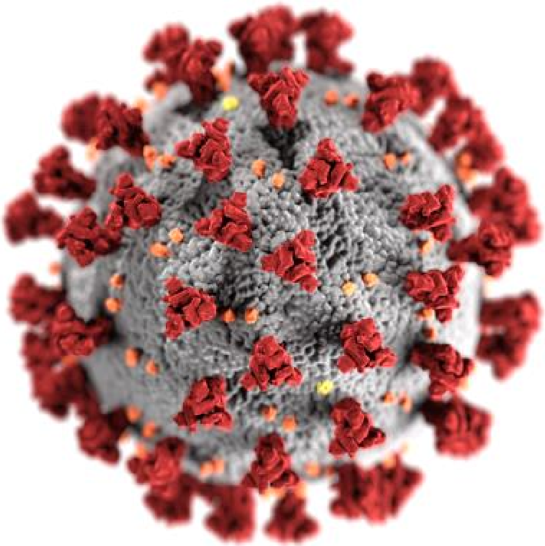
- For UK3 aggregate annual improvement rate between mid-2015 and mid-2019 was 1.2% p.a.
- Can compare with CMI model used for reserving.

Conclusions



Conclusions — I

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- Use all-cause data:
  - Portfolio cause-of-death data typically unavailable.
  - Cause-of-death coding incomplete in first shock.
- Use continuous-time methods to handle rapid changes in time.

Conclusions — II

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- Semi-parametric estimator for data exploration.
- Parametric age-period model for pricing or reserving.

Conclusions — III

Continuous age-period model:

Modelling by age

- Needs little flexibility.
- Use Hermite splines.

Modelling in time

- Needs lots of flexibility.
- Use Schoenberg [1964] splines.

Conclusions — IV

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Continuous age-period model:

- Add knots around pandemic shocks.
- Exercise judgement as to normal mortality level.
- Can estimate portfolio-specific improvement rate.
- Can even allow for unreported deaths.

Acknowledgements

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References I

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S. J. Richards. A Hermite-spline model of post-retirement mortality. *Scandinavian Actuarial Journal*, 2020:2:110–127, 2020. doi: 10.1080/03461238.2019.1642239.

References II

S. J. Richards. Mortality shocks and reporting delays in portfolio data. *Longevitas Ltd*, 2021a. URL https://www.longevitas.co.uk/site/library/Monitoring_mortality.pdf.

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Coronavirus graphic from CDC

References III

More on longevity risk at www.longevitas.co.uk

1 Legal matters

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