De Nederlandsche Bank, Amsterdam

Mis-estimation risk

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1. Questions to be answered

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- What uncertainty exists in our model for risk?
- What capital should we hold for that uncertainty?

2. Maximum likelihood

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- An asterisk (star) * indicates a true underlying (but unknown) value.
- A circumflex (hat) ˆ indicates an estimate.

2. Maximum likelihood

- We assume a log-likelihood function, $\ell(\underline{\theta})$, for a parameter vector $\underline{\theta}$.
- We seek the true underlying values, $\underline{\theta}^*$.
- What we *have* is the maximum-likelihood estimate (MLE), $\hat{\underline{\theta}}$.

2. Maximum likelihood

- We also seek the true underlying variance-covariance matrix for $\underline{\theta}$, V^*.
- What we *have* is the observed information matrix from $\ell(\underline{\hat{\theta}})$, I.
- I is matrix of negative second partial derivatives of ℓ evaluated at $\underline{\hat{\theta}}$.
- I is matrix formed from $-\dfrac{\partial^2}{\partial\theta_i\partial\theta_j}\ell(\underline{\hat{\theta}})$ for all i and j.
- V^* can be approximated with I^{-1}

2. Maximum likelihood

The maximum-likelihood theorem states the following:

$$\underline{\hat{\theta}} \sim \text{MVN}(\underline{\theta}^*, V^*)$$

where MVN denotes the multivariate Normal distribution.

Since we don't know $\underline{\theta}^*$ or V^*, we use the following approximation:

$$\underline{\hat{\theta}} \sim \text{MVN}(\underline{\hat{\theta}}, I^{-1})$$

3. Measuring mis-estimation risk

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- We value the portfolio repeatedly using parameters consistent with MVN$(\underline{\hat{\theta}}, I^{-1})$.
- We simulate consistent parameters with $\underline{\hat{\theta}} + A\underline{z}$.
- $\underline{z}$ is a vector of N(0,1) variates of the same length as $\underline{\hat{\theta}}$.
- A is the Cholesky decomposition of I^{-1}, i.e. the "square root" of I^{-1}.

3. Measuring mis-estimation risk

- After simulating and valuing n times we have a set of portfolio valuations, S.
- We estimate the 99.5th percentile of mis-estimation capital as follows:

$$\left(\frac{99.5^{\text{th}} \text{ percentile of } S}{\text{mean of } S} - 1 \right) \times 100\%$$

4. Example

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- Dutch pension scheme, $n = 1000$.

Model	**Reserve (millions of euros)**	**Mis-estimation capital at 99.5%**
Age*Gender	6,219	+1.7%
Age*(Gender+Size)	6,704	+2.6%

5. Conclusions

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- Mis-estimation risk can be handled statistically.
- Results dependent on model and portfolio's data.

LONGEVITAS

References

RICHARDS, S. J. **2013** *Mis-estimation risk: measurement and impact*, Longevitas Ltd