De Nederlandsche Bank, Amsterdam

Basics of mortality modelling

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1. q_x

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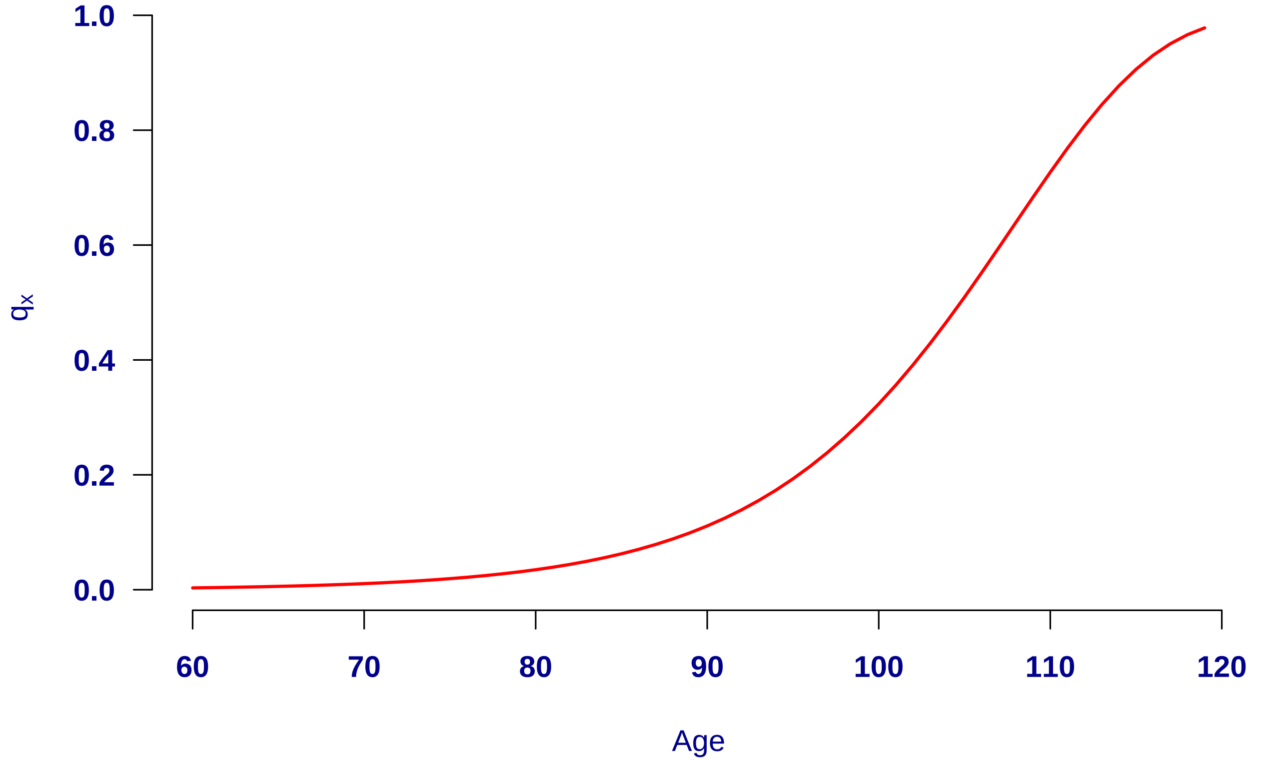
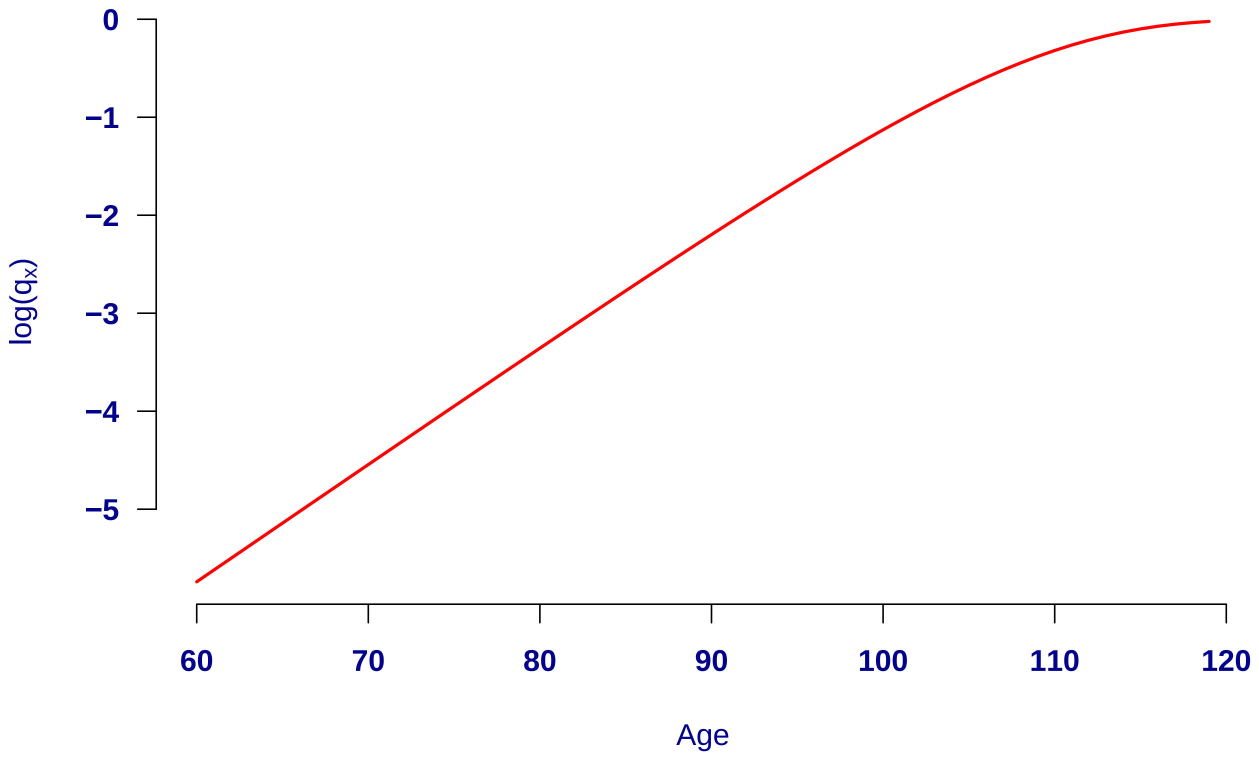
- $q_x = \Pr(\text{life aged exactly } x \text{ dies before age } x+1 | \text{alive at age } x)$.
- $p_x = \Pr(\text{life aged exactly } x \text{ survives to age } x+1 | \text{alive at age } x)$.
- $q_x, p_x \in [0, 1]$.
- x doesn't have to be an integer.

1. q_x



1. $\log(q_x)$



1. q_x

- So far we have considered death and survival over a single year.
- We now consider death and survival over an arbitrary, real-valued period of time...

1. q_x

- ${}_hq_x = \Pr(\text{life aged exactly } x \text{ dies before age } x+h | \text{alive at age } x)$.
- $h > 0$.
- $q_x = {}_1q_x$.

1. Drawbacks of q_x

— Loses precise information on time of death.

— Only uses complete (or potentially complete) years of exposure.

— Complicated adjustments needed to use fractional years of exposure.

— Hard to use when two or more decrements are present.

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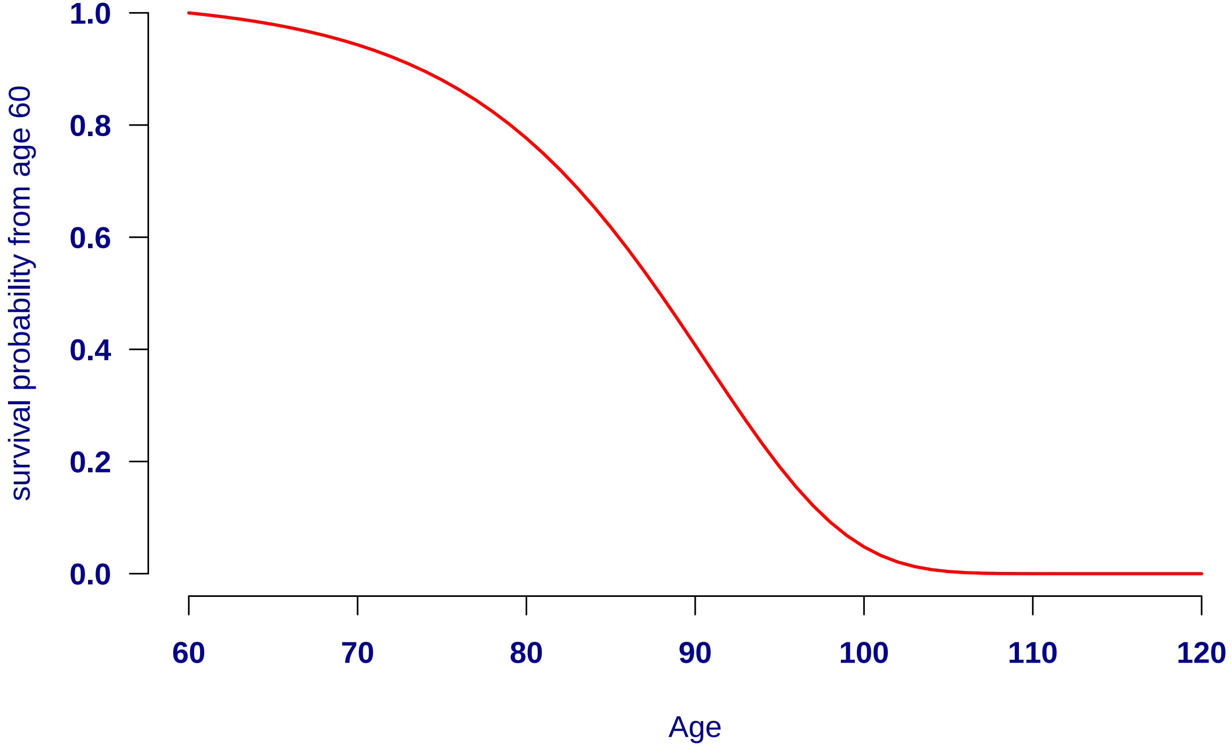
2. ${}_tp_x$

2. ${}_tp_x$

- ${}_tp_x = \Pr(\text{life aged exactly } x \text{ survives to age } x+t|\text{alive at age } x)$.
- $t > 0$.
- ${}_tp_x = 1 - {}_tq_x$.
- ${}_tp_x$ is known to statisticians as the *survival curve*.

2. ${}_tp_x$



1.0
0.8
0.6
0.4
0.2
0.0
survival probability from age 60
60
70
80
90
100
110
120
Age

3. μ_x

3. μ_x

- μ_x is the instantaneous force of mortality at age x.
- μ_x is the continuous-time analogue of q_x.
- $\mu_x > 0$.
- Statisticians call it the *hazard rate*.
- Engineers call it the *failure rate*.

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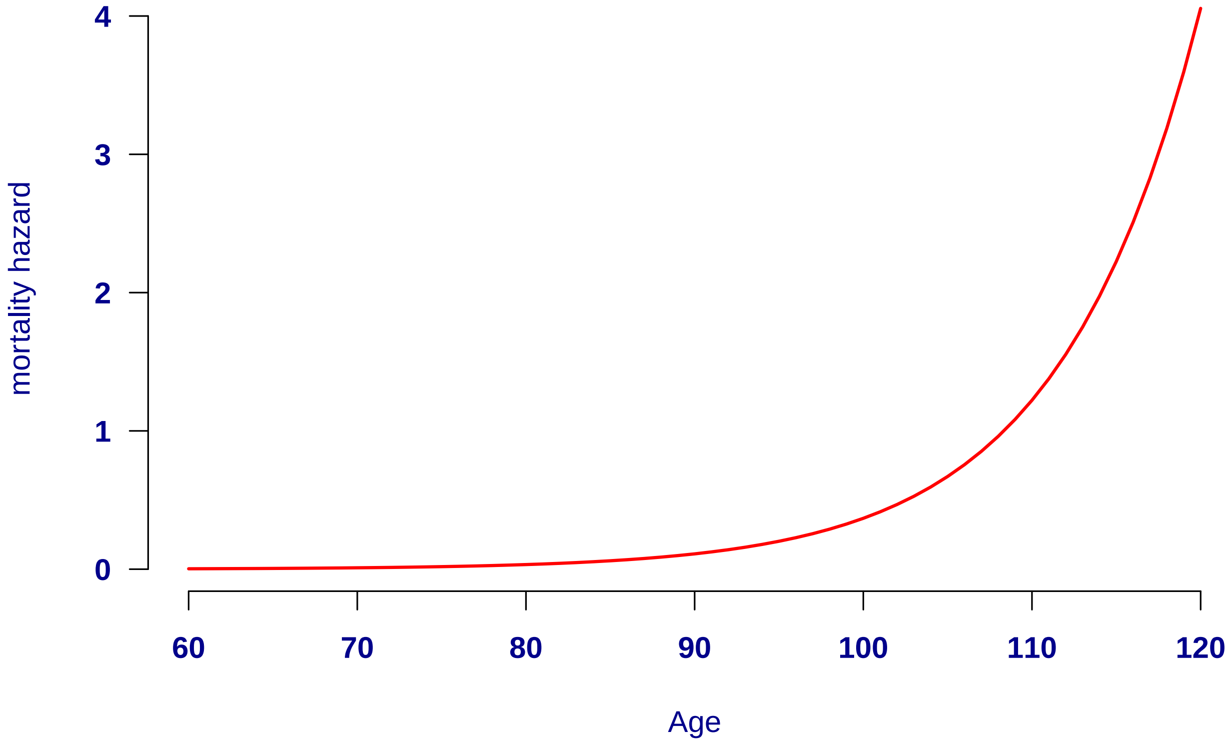
3. μ_x

$$\mu_x = \lim_{h \to 0^+} \frac{{}_h q_x}{h}$$

3. μ_x



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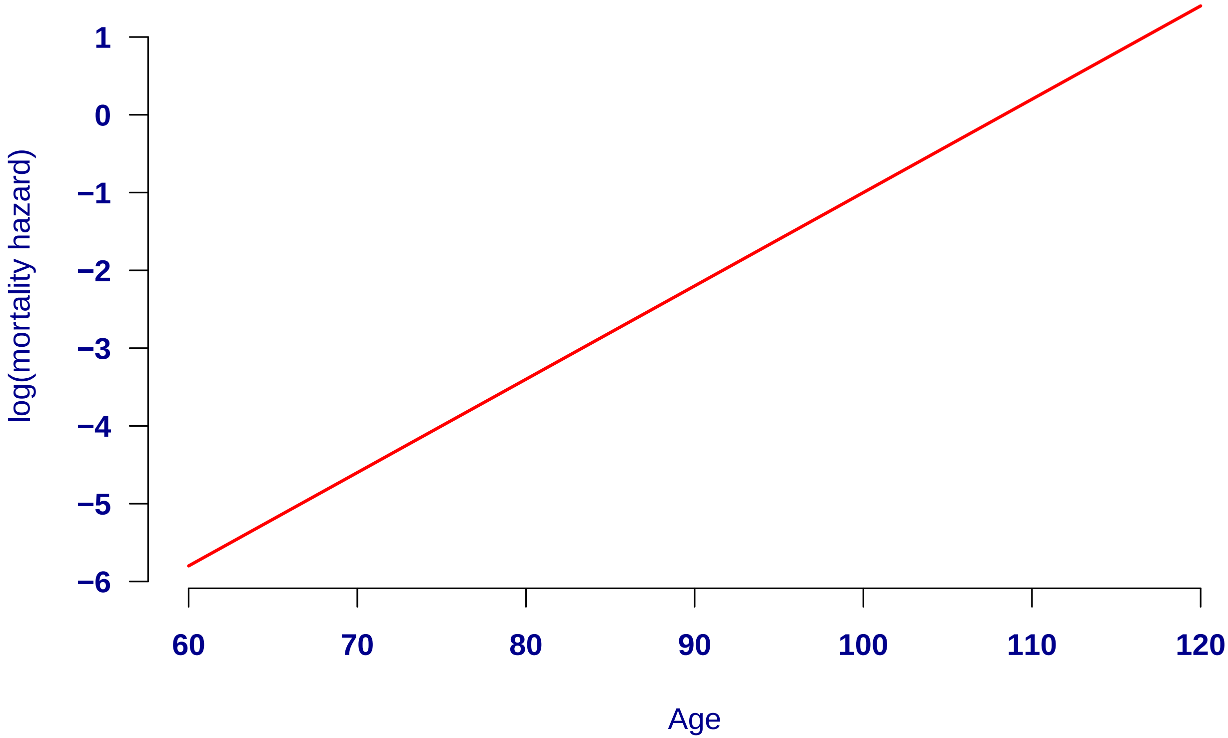
3. $\log(\mu_x)$



3. μ_x and ${}_tp_x$

$$ {}_tp_x = \exp\left(-\int_0^t \mu_{x+s} ds\right) $$

$H_x(t) = \int_0^t \mu_{x+s} ds$ is known as the *integrated hazard function.*

3. Advantages of using μ_x

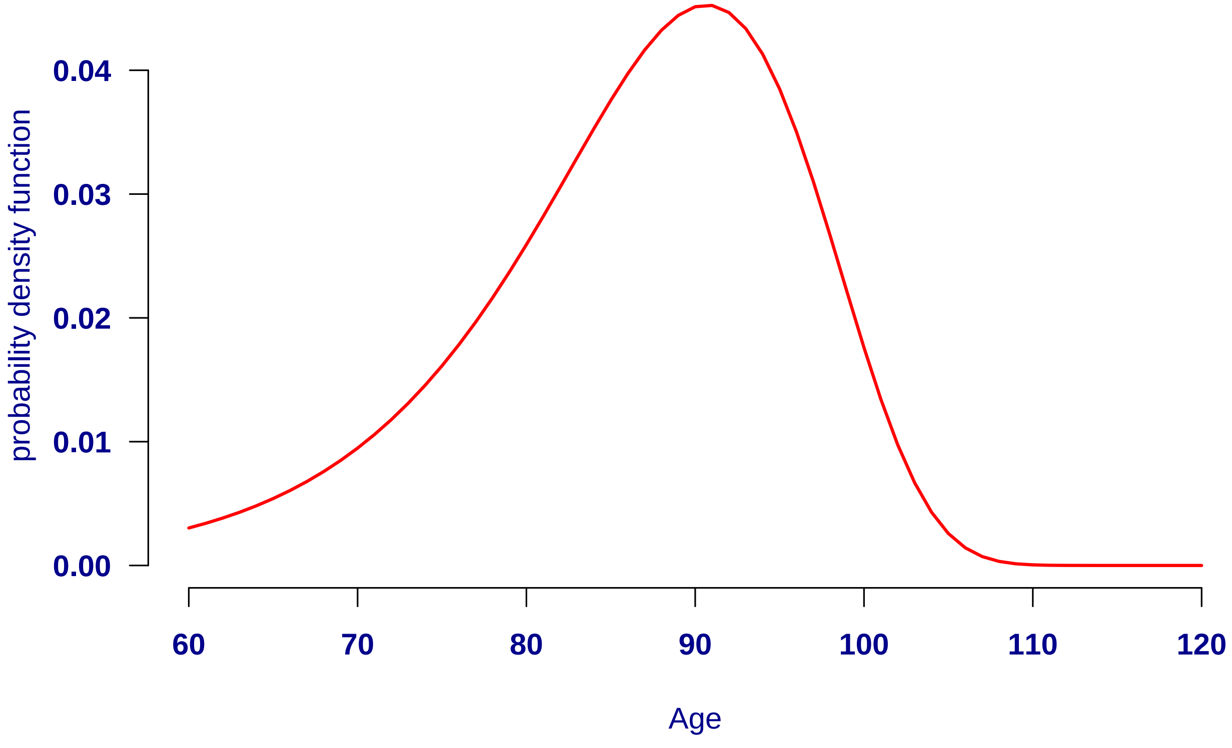
+ Uses all available data.
+ Can handle fractional years of exposure with ease.
+ Ideal if two or more competing decrements are present.
+ Lends itself to treating future lifetime as a random variable...

4. Lifetime as a random variable

4. Lifetime as a random variable

- T_x is the unknown future lifetime of a life aged x now.
- T_x is a continuous random variable.
- T_x has probability density function ${}_tp_x\mu_{x+t}, t > 0$.

3. ${}_tp_x\mu_{x+t}$



0.04
0.03
0.02
0.01
0.00
probability density function
60
70
80
90
100
110
120
Age

4. Lifetime as a random variable

- Mortality work is about *probabilities* and *probability distributions*.
- Tight relationship between μ_x and ${}_tp_x$.

5. Why use μ_x instead of q_x?

5. Why use μ_x instead of q_x?

- μ_x makes better use of available data.
- μ_x is easier to handle if there is more than one mode of exit.
- μ_x makes it easier to fit models to more than one year of data.
- With μ_x you can derive *every other mortality measure*.

6. Conclusions

5. Conclusions

- For mortality modelling we will use μ_x.