Longevity 18, London

Robust mortality forecasting in the presence of outliers

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Overview

1. Motivation
2. Univariate forecasts
3. Multivariate forecasts
4. 2D P-spline model
5. Conclusions

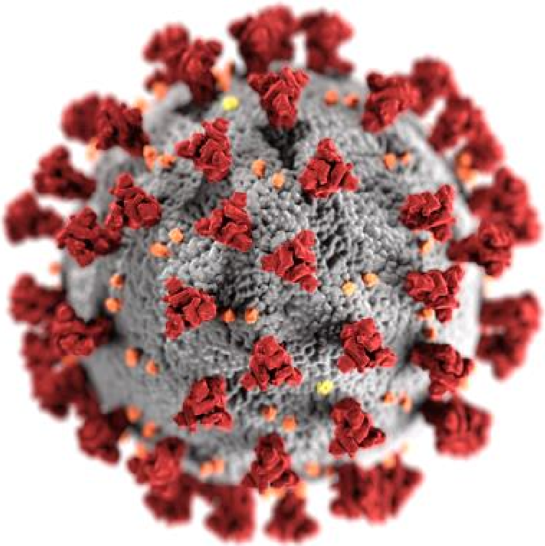
About this presentation

Fast introduction to robust mortality forecasting.

Further details in Richards [2023], freely available at:

www.longevitas.co.uk/robust-forecasting

1 Motivation



1 Three problems for actuaries

Covid-affected data cause:

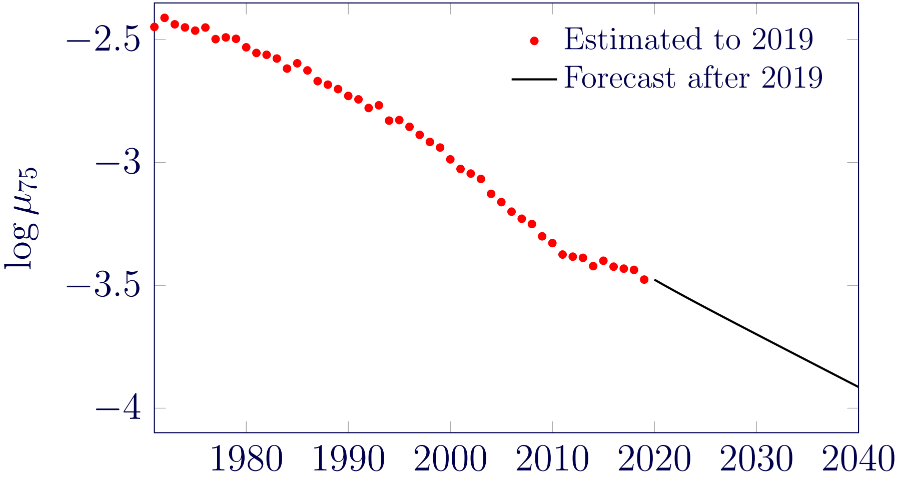
1. Broken forecasts.
2. Biased starting points.
3. Inflated variance.

1 Broken forecasts

Covid-19 distorts central projections…

Using data to 2019

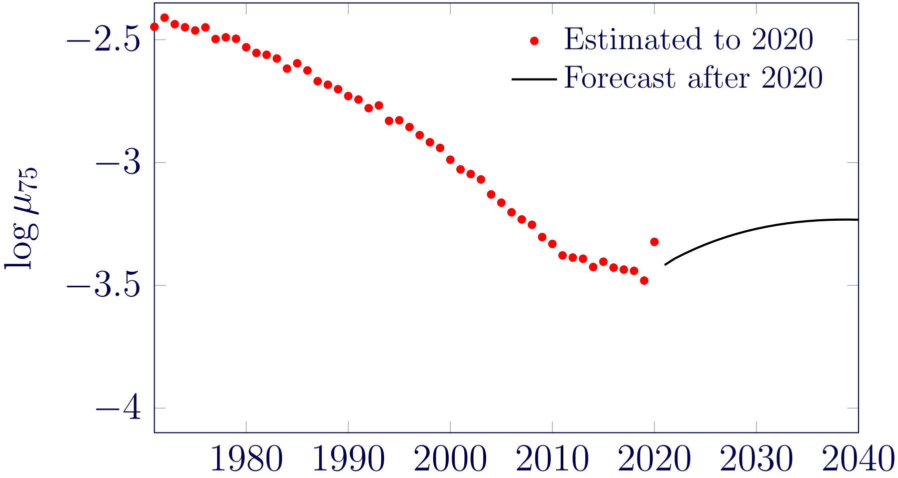
ARIMA forecast of time index in Lee-Carter model:



Source: Data for males in England & Wales, ages 50–105, 1971–2019.

Using data to 2020

ARIMA forecast of time index in Lee-Carter model:



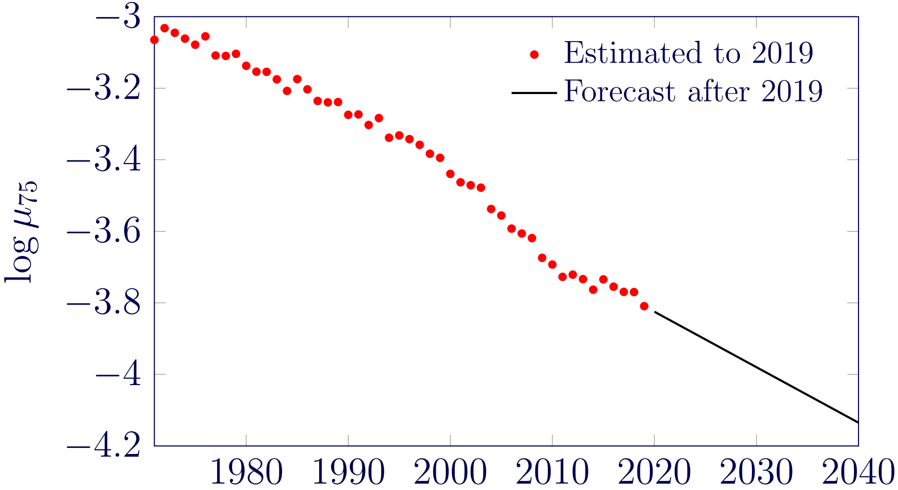
Source: Data for males in England & Wales, ages 50–105, 1971–2020.

1 Biased starting points

Covid-19 leads to biased starting points…

Using data to 2019

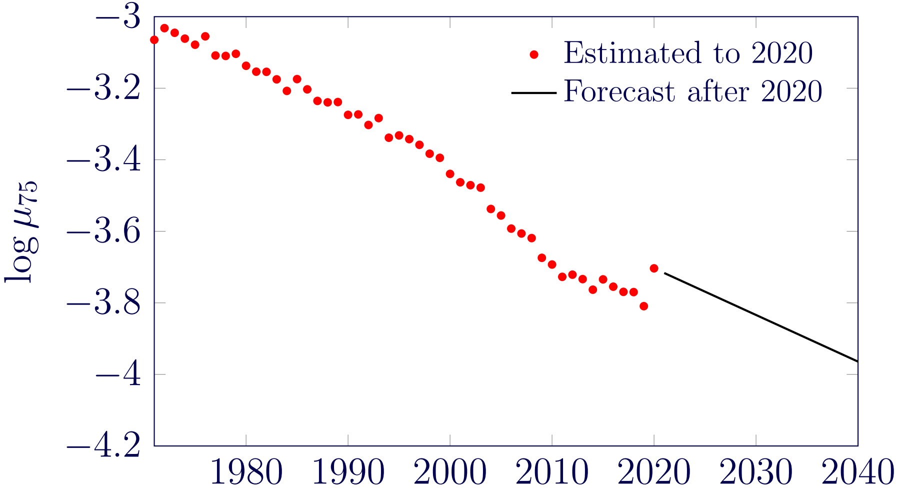
Bivariate random-walk forecast under M5 model:



Source: Data for females in England & Wales, ages 60–105, 1971–2019.

Using data to 2020

Bivariate random-walk forecast under M5 model:

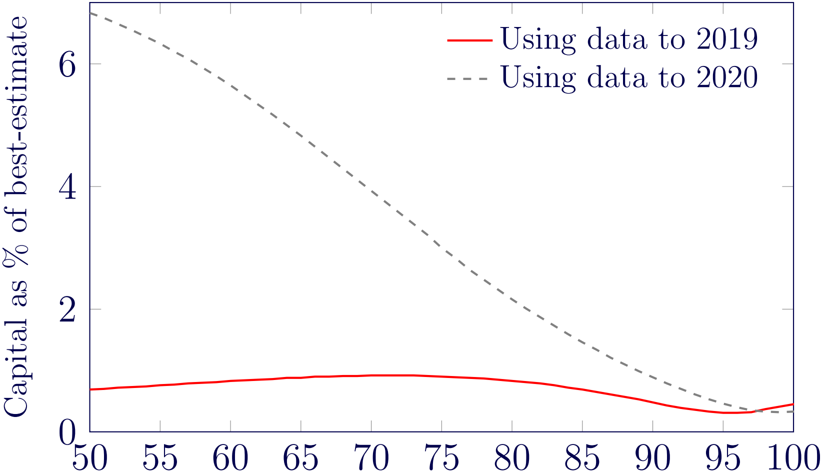


Source: Data for females in England & Wales, ages 60–105, 1971–2020.

1 Inflated variance

Outliers increase VaR capital requirements. . .

Value-at-risk capital at 99.5%



Source: 10,000 recalibrations of Lee-Carter model using data for males in England & Wales. Annuity cashflows discounted at 0% per annum.

1 Problem

- Covid-19 breaks forecasting models in three important ways.
- How can we robustify forecasts for actuarial tasks?

1 Robustification requirements

1. Remove distortion in parameter estimates.
2. Calculate "clean" starting points for forecasts.
3. Estimate variance robustly.
4. Need objective methodology for (1)-(3) to allow repeated recalibration under VaR-style simulations.

1 Solution

- Identify outliers with statistical tests.
- Co-estimate outlier effects with other parameters.

2 Univariate forecasts

2 Univariate forecasting

- A univariate model has multiple parameter vectors, but only one represents a time index to forecast.
- Example from Lee and Carter [1992]:

$$\log \mu_{x,y} = \alpha_x + \beta_x \kappa_y$$

- $\hat{\alpha}_x$ and $\hat{\beta}_x$ are held constant in the forecast.
- An ARIMA model is fitted to the $\hat{\kappa}_y$ time index to forecast the trend.

2 Outlier definition

Outlier

An observation that is further from the one-year-ahead forecast than is consistent with the noise variance.

2 Univariate outliers

To robustify an ARIMA model, Chen and Liu [1993]:

1. Proposed test statistics to identify outliers.
2. Proposed further test statistics to *classify* outliers.

2 Four outlier types

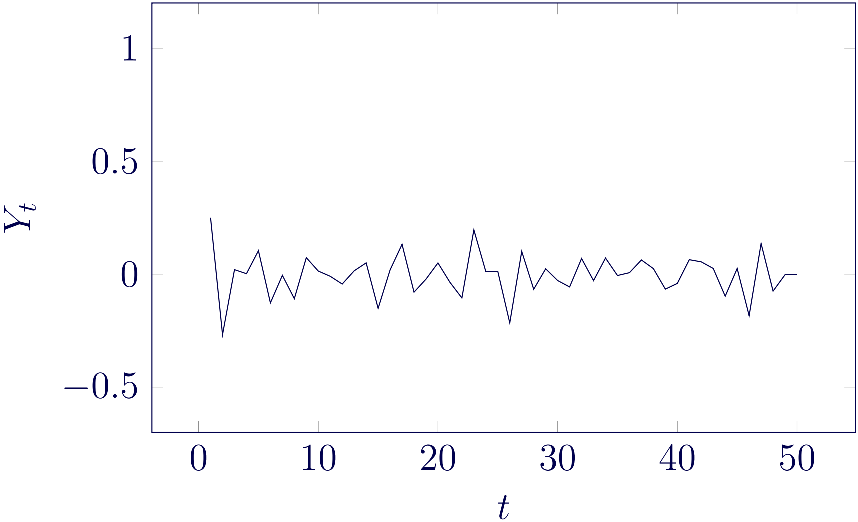
IO	Innovation outlier
AO	Additive outlier
TC	Temporary change
LS	Level shift

2 Illustrating outlier types

Consider a moving-average (MA) process:

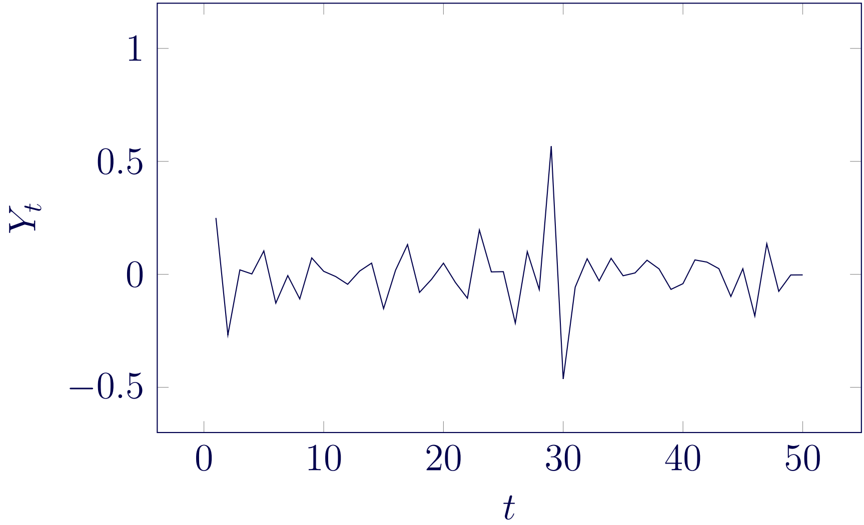
$$Y_t = \epsilon_t - 0.8\epsilon_{t-1} \tag{1}$$

2 Uncontaminated MA process



Source: Richards [2023, Figure 3].

2 IO — Innovation Outlier



Source: Richards [2023, Figure 3].

2 Innovation outlier

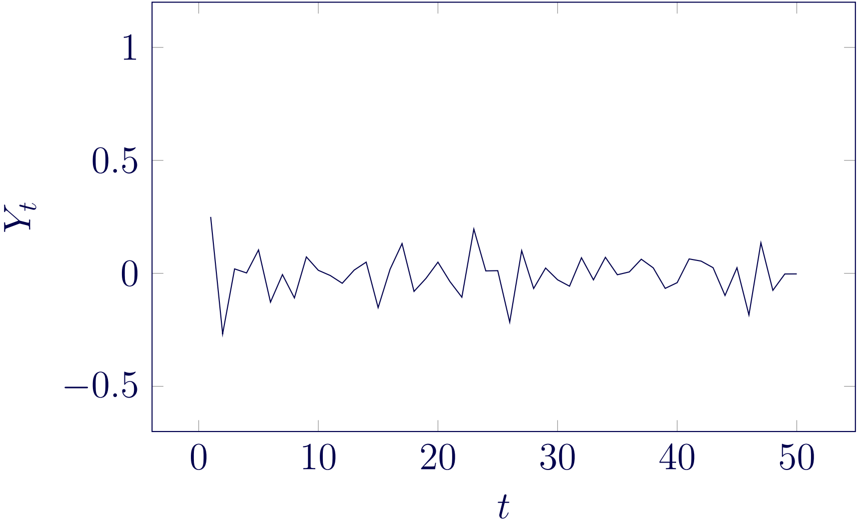
A modest outlier that is nevertheless integrated into the process.

Example

A year with heavy winter mortality due to influenza, possibly with lighter mortality the following year.

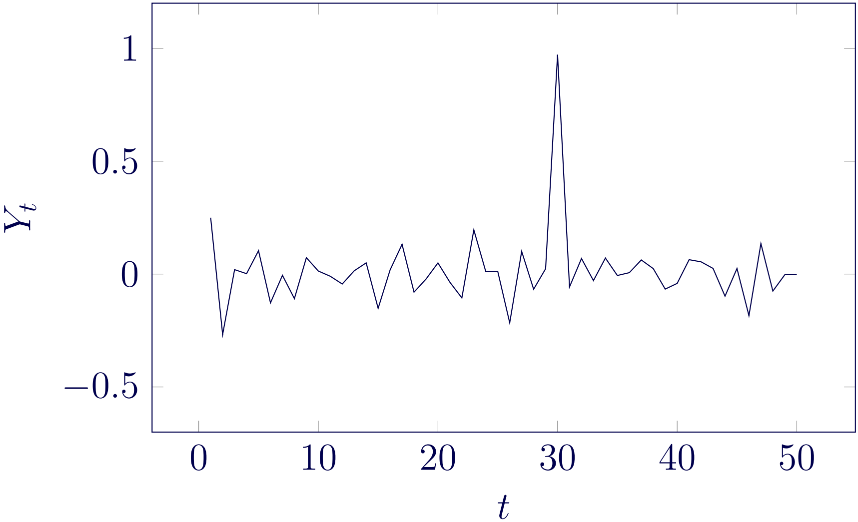
Handling: leave alone.

2 Uncontaminated MA process



Source: Richards [2023, Figure 3].

2 AO — Additive Outlier



Source: Richards [2023, Figure 3].

2 Additive outlier

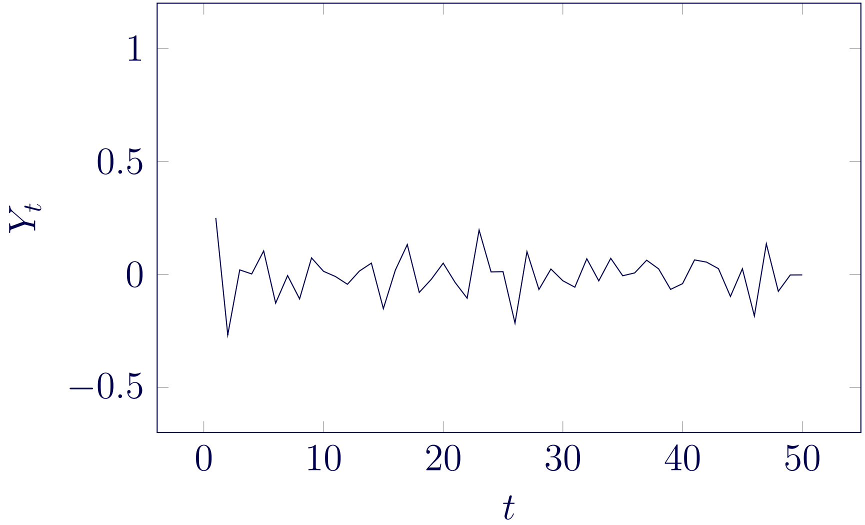
A more extreme outlier that is not integrated into the process.

Example

War or pandemic in a single year.

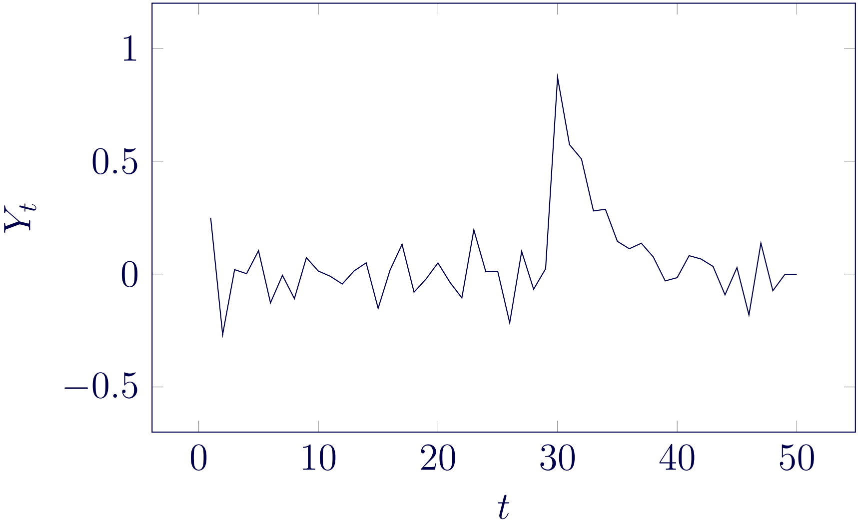
Handling: co-estimate the outlier effect to remove bias in other parameters.

2 Uncontaminated MA process



Source: Richards [2023, Figure 3].

2 TC — Temporary Change



Source: Richards [2023, Figure 3].

2 Temporary change

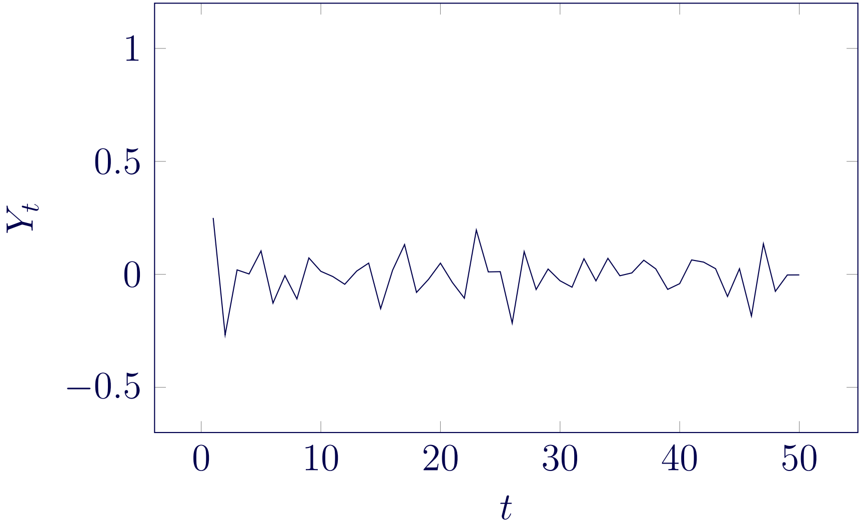
Two or more consecutive outliers that are not integrated into the process.

Example

War or pandemic spread over more than one year.

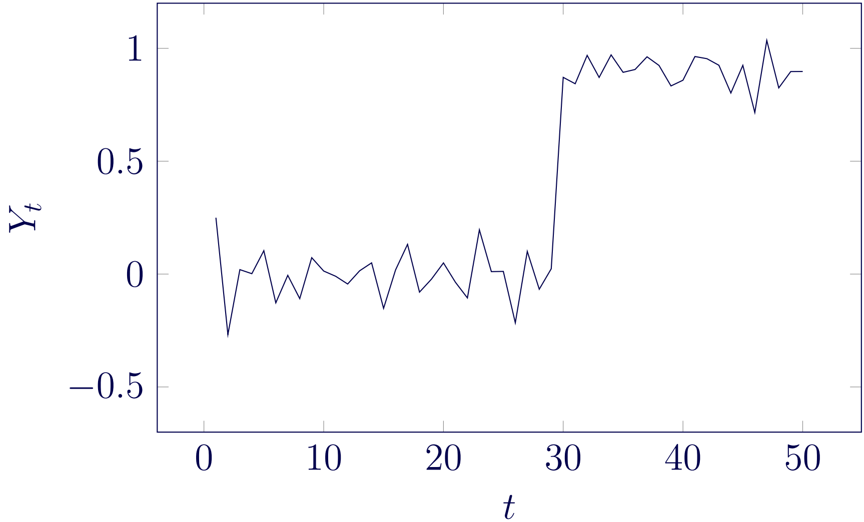
Handling: co-estimate the outlier effects to remove bias in other parameters.

2 Uncontaminated MA process



Source: Richards [2023, Figure 3].

2 LS — Level Shift



Source: Richards [2023, Figure 3].

2 Level shift

Permanent change in level of process.

Example

After German reunification in 1990, old-age mortality in East converged rapidly on levels in the West [Grigoriev et al., 2021].

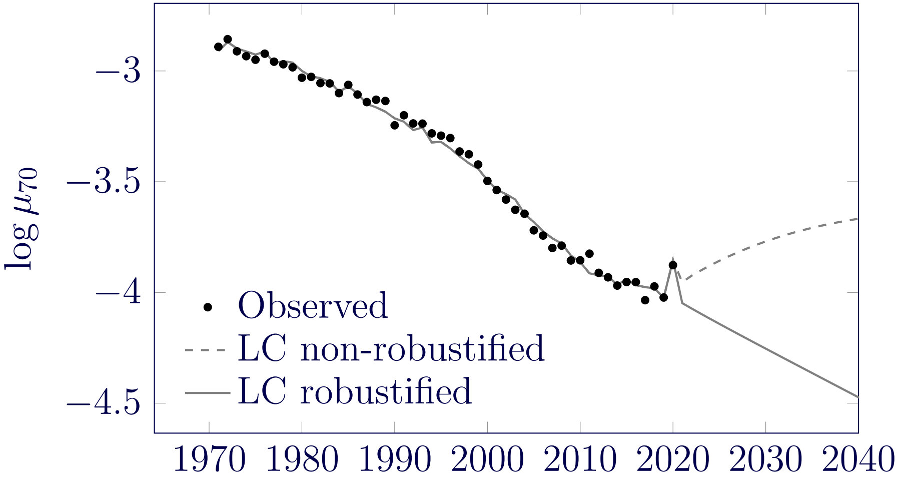
Handling: review model or data period.

2 Chen and Liu [1993]

Note

An outlier can be detected anywhere, but "it is impossible to empirically distinguish the type of an outlier at the very end of a series" [Chen and Liu, 1993, page 286].

2 Robust Lee-Carter forecast



Source: Richards [2023, Figure 4(b)].

3 Multivariate forecasts

3 Multivariate forecasting

- A multivariate model has two or more time indices.
- Example from Cairns et al. [2006]:

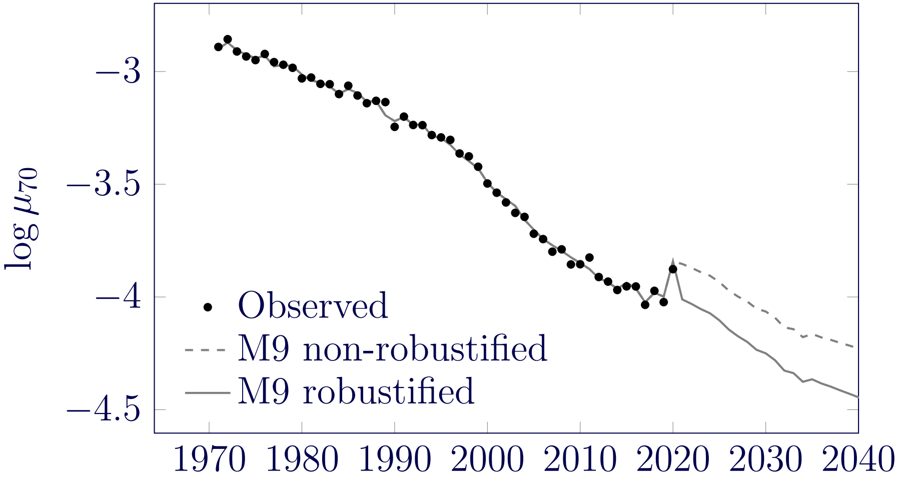
$$\log \mu_{x,y} = \kappa_{0,y} + (x - \bar{x})\kappa_{1,y}$$

- $\hat{\kappa}_{0,y}$ and $\hat{\kappa}_{1,y}$ are forecast jointly as a bivariate random walk with drift.

3 Multivariate robustification

- Use approach of Galeano et al. [2006].
- Robustify first differences [Richards, 2023, Appendix C].

3 Robust M9 forecast



Source: Richards [2023, Figure 9(b)].

3 Other multivariate options

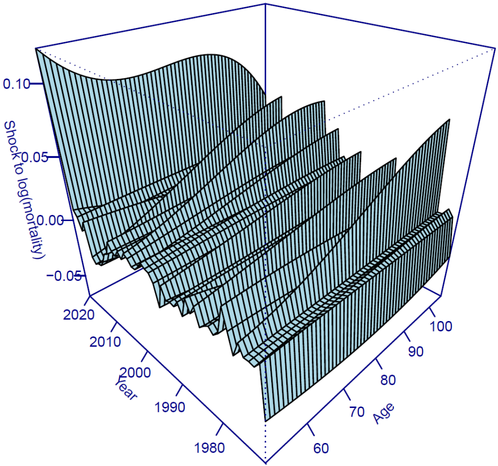
Other approaches to identifying outliers in multivariate data, e.g. Hadi [1994].

4 2D P-spline model

4 2D P-spline model

- Introduced by Currie et al. [2004].
- Extended by Kirkby and Currie [2010] to estimate period shocks. . .

4 Period shocks (EW males)

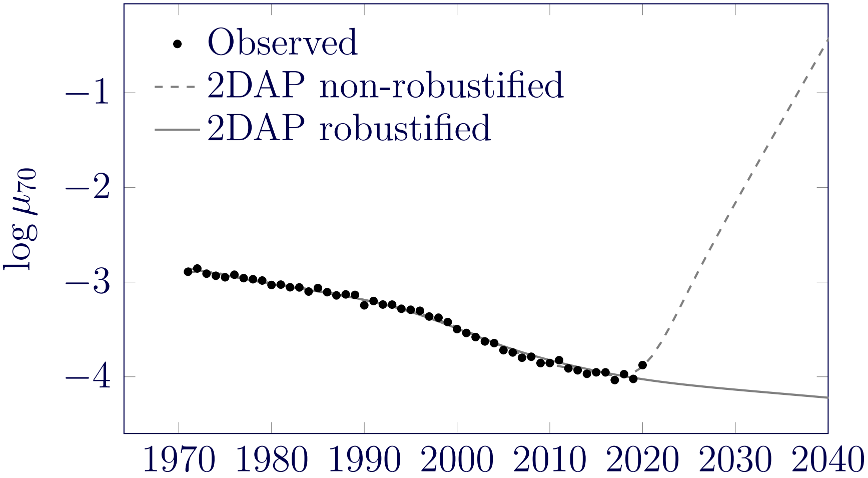


Source: Richards [2023, Figure 12].

4 2D P-spline model

- Forecast using *penalty function*.
- Estimating period shocks robustifies penalty forecast. . .

4 Robust 2DAP forecast



Source: Richards [2023, Figure 13(b)].

5 Conclusions

5 Conclusions — I

Co-estimation of outliers and parameters:

1. Reduces bias in forecasting parameters.
2. Yields better starting points for forecasts.
3. Reduces variance in capital requirements.

5 Conclusions — II

Univariate forecasting

- Lee-Carter and APC models.
- Use approach of Chen and Liu [1993].

Multivariate forecasting

- Cairns-Blake-Dowd & Tang-Li-Tickle models.
- Use approach of Galeano et al. [2006].

2D P-spline

- Currie-Durban-Eilers model.
- Use approach of Kirkby and Currie [2010].

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S. J. Richards. Robust mortality forecasting in the presence of outliers. Longevitas working paper, 2023.

Coronavirus graphic from CDC

More on robust forecasting at www.longevitas.co.uk/robust-forecasting

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