Longevity 21, Rome

COVID-19 in mortality forecasts

Stephen J. Richards

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Overview

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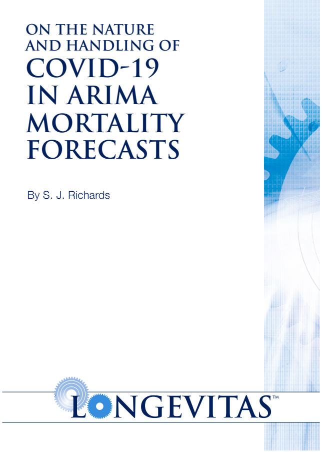
Foreword


ON THE NATURE AND HANDLING OF
COVID-19 IN ARIMA MORTALITY FORECASTS

By S. J. Richards

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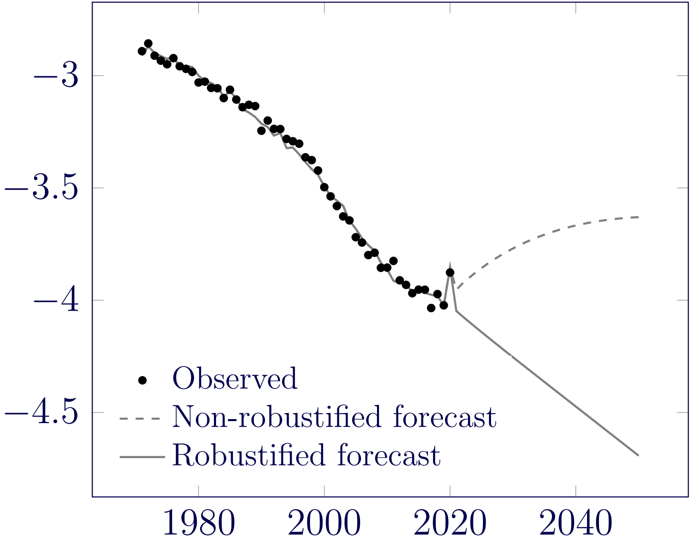
PDF available at: https://www.longevitas.co.uk/published-paper/nature-and-handling-covid-19-arima-mortality-forecasts

1 The COVID-19 problem

COVID-19 forecasting problem



Source: Richards [2024, Figure 4(b)]. Data for males in England & Wales, 1971-2020.

COVID-19 bias problem

Q Why don't we just do nothing and let the problem fade into history?

A COVID-19 biases parameter estimates, wherever it sits in the data.

COVID-19 bias problem

- $\hat{\sigma}^2$ is the estimate of volatility in κ_y.
- $\hat{\sigma}^2$ drives VaR capital requirements in Lee-Carter model [Kleinow and Richards, 2016].
- $\hat{\sigma}^2 = 1.05 \times 10^{-4}$ without handling 2020 outlier.
- $\hat{\sigma}^2 = 5.18 \times 10^{-5}$ allowing for 2020 outlier.
- $\Rightarrow$ allowing for outlier halves the parameter estimate that drives VaR capital requirements.

Source: $\hat{\sigma}^2$ estimates from Richards [2024, Figures B.5 and B.6].

2 Forecasting models

Lee-Carter model

Let $\mu_{x,y}$ denote the mortality hazard at age x in year y.

Lee and Carter [1992] proposed a log-bilinear model:

$$\log \mu_{x,y} = \alpha_x + \beta_x \kappa_y$$

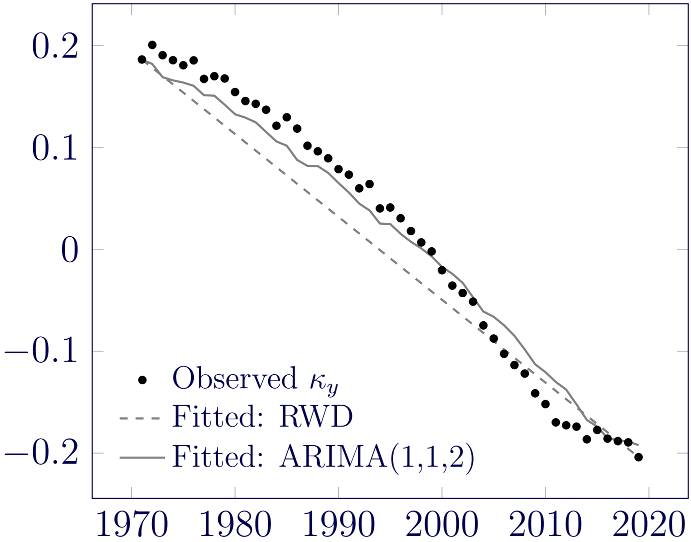
with two identifiability constraints.

Lee-Carter forecasting

- Forecast mortality by projecting κ_y.
- Lee & Carter used a random walk with drift…

  …but ARIMA models often fit better.

ARIMA models fit better



Source: own calculations using κ_y estimates for males in England & Wales, 1971-2019.

Random walk with drift

$$\Delta\kappa_y - \mu = \epsilon_y$$

where:

- Δ is the backward difference operator,
- μ is the constant annual drift term, and
- $\{\epsilon_y\}$ is a collection of uncorrelated innovations with mean zero and constant variance σ^2.

ARIMA(p, 1, q) model

$$\phi(B)\left[\Delta\kappa_y - \mu\right] = \theta(B)\epsilon_y$$

where:

- B is the backshift operator, i.e. $B\kappa_y = \kappa_{y-1}$,
- $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$, and
- $\theta(B) = 1 + \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$.

Forecasting t steps ahead

Random walk with drift:

$$\kappa_{y+t} = \underbrace{\kappa_y + t\mu}_{\substack{\text{linear}\\\text{trend}}} + \sum_{i=1}^{t} \epsilon_{y+i}$$

ARIMA(p, 1, q) model:

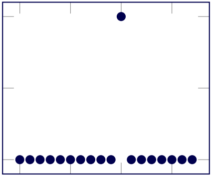
$$\kappa_{y+t} = \underbrace{\kappa_y + t\mu}_{\substack{\text{linear}\\\text{trend}}} + \underbrace{\sum_{i=1}^{t} \frac{\theta(B)}{\phi(B)} \epsilon_{y+i}}_{\substack{\text{cumulative}\\\text{stochastic}\\\text{deviation}}}$$

Source: Richards [2026, Section 5].

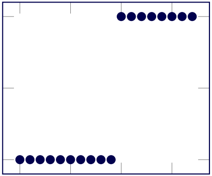
3 Outliers

Types of outlier

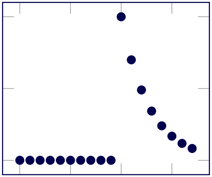
Additive Outlier (AO)



Level Shift (LS)



Temporary Change (TC)



Source: Richards [2026, Figure 7].

Outlier detection

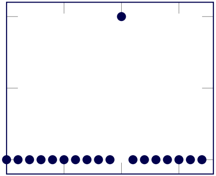
Chen and Liu [1993] proposed test statistics to:

1. Identify location of outliers, and
2. Classify type of outlier.

Note: The type cannot be determined for an outlier at the end of a series.

Types of outlier

Additive Outlier (AO)



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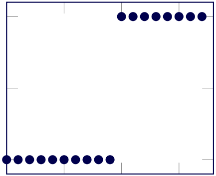

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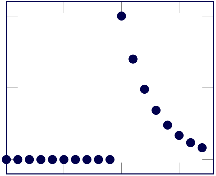
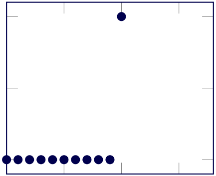
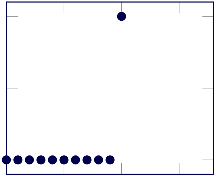


Types of outlier

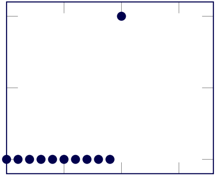
Additive Outlier (AO)



Level Shift (LS)



Temporary Change (TC)



Outliers

Let κ^*_{y+t} denote κ_{y+t} contaminated by m outliers:

$$\kappa^*_{y+t} = \kappa_{y+t} + \underbrace{\sum_{j=1}^{m} \omega_j I_j(y+t)}_{\text{outlier component}}$$

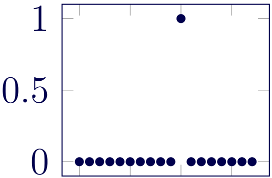
where:

- ω_j is the size of outlier j, and
- $I_j(y+t)$ is the relative effect of outlier j at $y+t$.

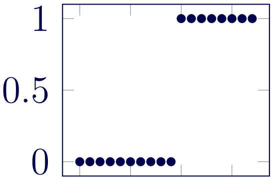
Source: Richards [2026, Section 6].

Value of I_j

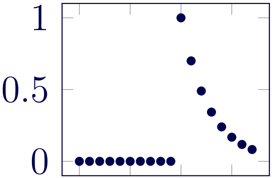
Additive Outlier (AO)



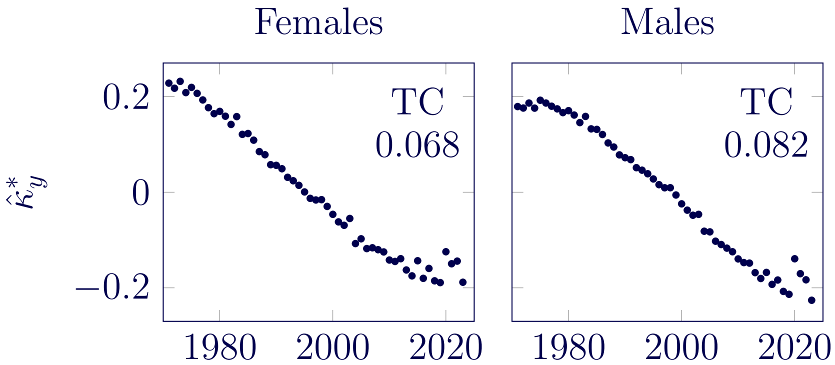
Level Shift (LS)



Temporary Change (TC)

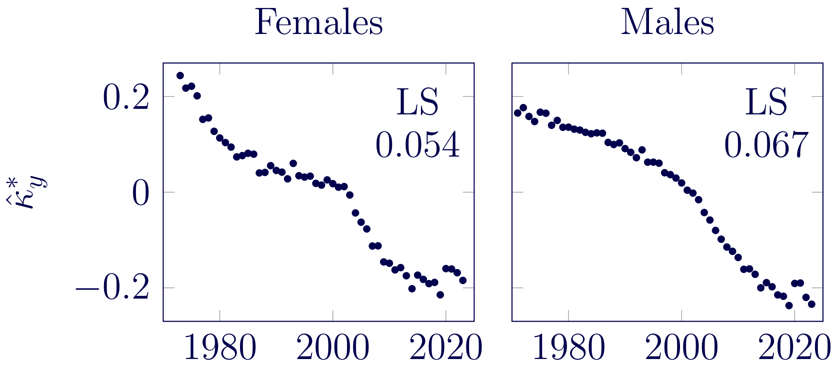


2020 outliers: Italy



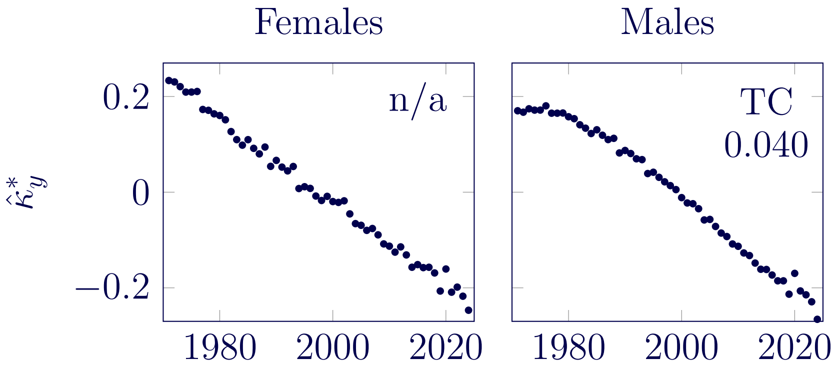
Source: Richards [2026, Figure 3 and Table 3].

2020 outliers: Netherlands



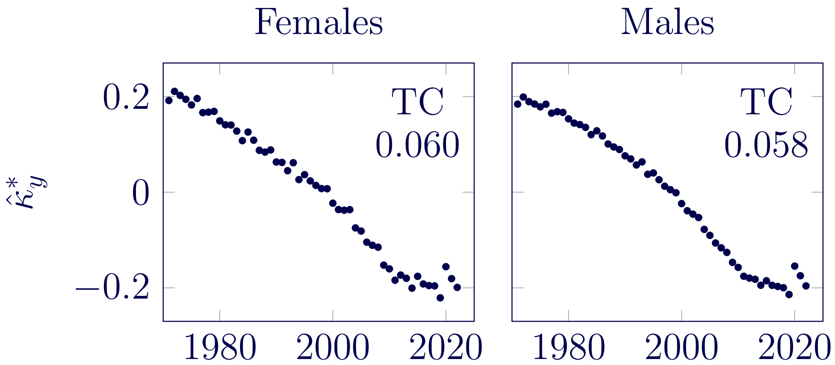
Source: Richards [2026, Figure 4 and Table 3].

2020 outliers: Sweden



Source: Richards [2026, Figure 5 and Table 3].

2020 outliers: UK



Source: Richards [2026, Figure 6 and Table 3].

Outliers in an ARIMA model

Random walk with drift:

$$\Delta\kappa_y - \mu = \epsilon_y$$

ARIMA model:

$$\phi(B)\left[\Delta\kappa_y - \mu\right] = \theta(B)\epsilon_y$$

ARIMA model with m outliers:

$$\phi(B)\left[\Delta\kappa_y^* - \mu - \Delta\sum_{j=1}^{m}\omega_j I_j(y)\right] = \theta(B)\epsilon_y$$

Forecasting with outliers

Forecasting t steps ahead:

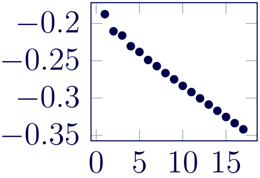
$$\kappa^*_{y+t} = \underbrace{\kappa_y + t\mu}_{\substack{\text{linear}\\ \text{trend}}} + \underbrace{\sum_{j=1}^{m} \omega_j I_j(y+t)}_{\text{outlier component}} + \underbrace{\sum_{i=1}^{t} \frac{\theta(B)}{\phi(B)} \epsilon_{y+i}}_{\substack{\text{cumulative}\\ \text{stochastic}\\ \text{deviation}}}$$

Source: Richards [2026, Section 6].

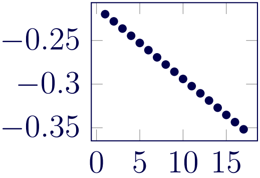
Forecast components

ARIMA(1,1,0) central forecast of κ^*_{2023+t} for females in Italy.

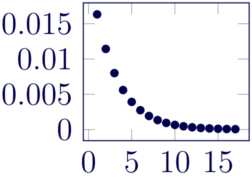
(a) κ^*_{2023+t} forecast



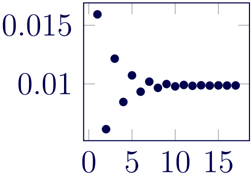
(b) Linear trend



(c) TC outlier



(d) Cum. stoch. dev.



Source: Richards [2026, Figure 9].

4 Conclusions

4 Conclusions

1. ARIMA models fit data better than a random walk with drift.
2. Methodology of Chen and Liu [1993] identifies position and type of outliers.
3. COVID-19 caused lasting mortality effects in many countries.
4. Forecast components:
    - 4.1 Linear trend,
    - 4.2 Ongoing outlier effects, and
    - 4.3 Cumulative stochastic deviation.

References I

C. Chen and L-M. Liu. Joint estimation of model parameters and outlier effects in time series. *Journal of the American Statistical Association*, 88(421): 284–297, 1993. doi: 10.2307/2290724.

T. Kleinow and S. J. Richards. Parameter risk in time-series mortality forecasts. *Scandinavian Actuarial Journal*, 2016(10):1–25, 2016. doi: 10.1080/03461238.2016.1255655.

R. D. Lee and L. Carter. Modeling and forecasting US mortality. *Journal of the American Statistical Association*, 87:659–671, 1992. doi: 10.1080/01621459.1992.10475265.

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S. J. Richards. Robust mortality forecasting in the presence of outliers. *British Actuarial Journal*, 29: e19, 2024. doi: 10.1017/S1357321724000175.

S. J. Richards. On the nature and handling of COVID-19 in ARIMA mortality forecasts. *Longevitas Ltd*, 2026.

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