

Heriot-Watt University, Edinburgh

The actuarial origins of survival models

Stephen J. Richards

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- Edinburgh software firm founded 2006.
- Actuarial tools for mortality analysis and projection.
- Clients in UK, USA, Canada and Switzerland.

THE ACTUARIAL ORIGINS OF SURVIVAL MODELS

By S. J. Richards



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For a life aged x , the survival function to age $x + t$ is:

$${}_tp_x = \exp \left(- \int_0^t \mu_{x+s} ds \right) \quad (1)$$

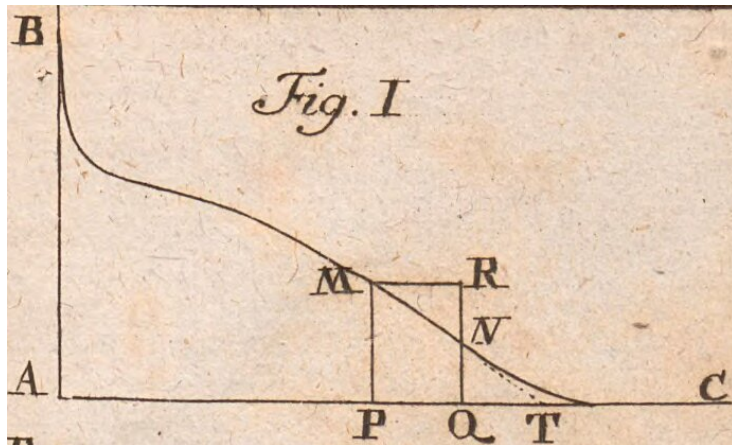
$$= e^{-H_x(t)} \quad (2)$$

where:

- μ_{x+s} is the *mortality hazard* at age $x + s$, and
- $H_x(t)$ is the *integrated hazard function*.

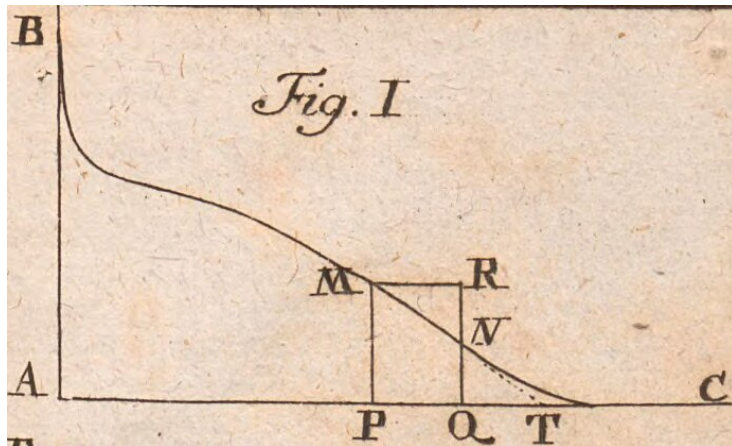
$H_x(t)$ was pivotal for actuaries in the 19th century.

Lambert [1772] worked with the survival curve:



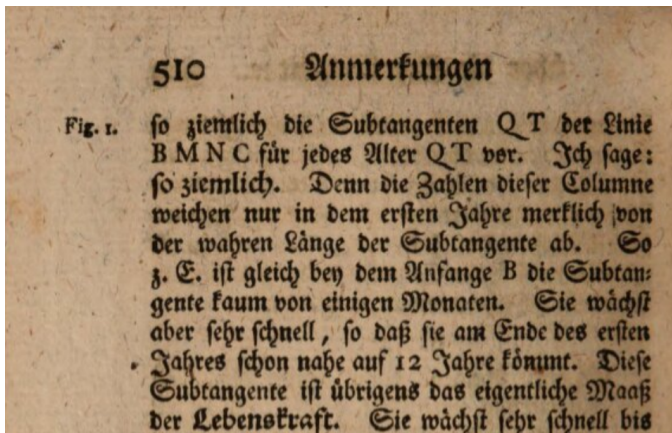
Source: Lambert [1772, Figure I].

Length of QT is $1/\mu_Q$, i.e. the inverse hazard at point N :



Source: Lambert [1772, Figure I].

Lambert called the sub-tangent QT the *Lebenskraft* (life-force):



2 Gompertz

- English actuary.
- Gompertz [1825] proposed the earliest realistic model for the mortality hazard:

$$\mu_x = e^{\alpha + \beta x}$$

(modern notation!)

Reading Gompertz [1825] is a challenge:

$$\lambda C = \lambda \left(\overset{1,05^{-1}}{\underset{1}{1}} \left[b \right] \right) + \lambda(1,05) - \lambda \left(\overset{1,05^{-1}}{\underset{0}{1}} \left[b \right] \right)$$

which is easier to follow when put in modern notation:

$$\log C = \log a_b^{5\%} + \log 1.05 - \log \ddot{a}_b^{5\%} \quad (3)$$

Source: Gompertz [1825, p.539]

Gompertz worked in *continuous time*:

“infinitely small intervals of time”

Gompertz [1825, p.518]

... and used the *mortality hazard*:

“intensity of [...] mortality”

Gompertz [1825, p.518]

- Gompertz didn't work directly with μ_x .
- Instead, he used the *survival function*, ${}_tp_x \dots$
 $\dots$ and the *integrated hazard*, $H_x(t)$.

For Gompertz's model:

$$H_x(t) = \frac{(e^{\beta t} - 1)}{\beta} e^{\alpha + \beta x}. \quad (4)$$

Gompertz noted that:

$$\frac{\log {}_t p_{x+t}}{\log {}_t p_x} = \frac{H_{x+t}(t)}{H_x(t)} = e^{\beta t}, \quad (5)$$

and therefore that:

$$\beta = \frac{1}{t} \log \left(\frac{\log {}_t p_{x+t}}{\log {}_t p_x} \right). \quad (6)$$

1. Pick x and $x + 2t$ to span the range of reliable data.
2. Estimate β *in a single step*.
3. Estimate α knowing β .

Gompertz's motivations were entirely practical:

1. Both a model and a model-fitting procedure,
2. Minimise calculation steps,
3. Interpolation of mortality at any age,
4. Closed-form expression for ${}_tp_x$, and
5. Closed-form expression for ${}_tp_{x:y}$.

- English actuary.
- Makeham [1860] proposed an improved model for mortality:

$$\mu_x = e^{\epsilon} + e^{\alpha + \beta x}$$

(modern notation!)

- e^{ϵ} described as “a certain uniform quantity” (p.303).

Q: Why did it take 35 years just to add a constant?

A: Actuaries also needed a fitting algorithm.

Makeham worked with the mortality hazard:

“force of mortality”

Makeham [1867, p.327]

Makeham's integrated hazard is:

$$H_x(t) = te^{\epsilon} + \frac{(e^{\beta t} - 1)}{\beta} e^{\alpha + \beta x}. \quad (7)$$

Makeham noted that:

$$\frac{\log {}_t p_{x+2t} - \log {}_t p_{x+t}}{\log {}_t p_{x+t} - \log {}_t p_x} = \frac{H_{x+2t}(t) - H_{x+t}(t)}{H_{x+t}(t) - H_x(t)} \quad (8)$$

$$= e^{\beta t} \quad (9)$$

and therefore that:

$$\beta = \frac{1}{t} \log \left(\frac{\log {}_t p_{x+2t} - \log {}_t p_{x+t}}{\log {}_t p_{x+t} - \log {}_t p_x} \right). \quad (10)$$

1. Pick x and $x + 3t$ to span the range of reliable data.
2. Estimate β in a single step.
3. Estimate ϵ in a single step.
4. Estimate α knowing β .

- English actuary.
- Wrote textbook for Institute of Actuaries [King, 1887].
- Extended Makeham's fitting procedure to make more use of available data[†].
- Still working with survival probabilities and $H_x(t)$.

[†] See Richards [2026, Section 9] for details.

5 Social context

“British society [...] was open to commercial enterprise [...] and had a growing middle class with wealth (and female relatives) to protect”

Macdonald [2026, p.4]

- Married Women's Property Act 1882.
- Prior to this, a married woman had no legal personality separate from her husband ('coverture').
- Parents insured the life of the father or son-in-law to provide financial security for a married daughter.
- Insurance policies written in trust were protected from a husband's creditors.



- German actuaries followed actuarial developments in Britain.
- They even contributed to British actuarial publications.

Noch ein merkwürdiger Versuch, die Zahl der Lebenden, aber nur von einem gewissen Alter ab, darzustellen, rührt von B. Gompertz her. ¹⁾ Er giebt für die Zahl der Lebenden

$$y = a \cdot m^{(nx)}$$

wo a , m , n Constanten sind, welche durch die Beobachtungen zu bestimmen sind. Die Zahl der Geborenen wird dabei, wie man sieht, gleich am angenommen.

Source: Moser [1839, p.279].

Wirkliche Erfahrungen aus einem ausgedehnten und mit unsrer Witwenkasse wohl ungefähr auf gleiche Linie zu stellenden Kreise finde ich in PRICE Observations on reversionary payments, S. 79. 269. 276 (nach der dritten Ausgabe dieses Werks von 1773), wo für die Gesammtheit der Pfarrer und Professoren in Schottland nach 17jährigem Durchschnitt die Zahl der stehenden Ehen zu 667, und die der Witwen

Source: Gauss [1873, p.145].

diejenige, die zur Ausgleichung der Sterblichkeitsverhältnisse der Hamburgischen Pensionskasse und der Verfasser des gegenwärtigen Gutachtens zur Bearbeitung der Sterblichkeitserfahrungen der Gothaer Bank angewandt hat (Masius' Rundschau, Jahrgang 1884, S. 309 und ff.), sondern eine wesentlich vereinfachte, die jüngst von King empfohlen wurde (The Institute of Actuaries', Text-Book, Part II), und die ebenfalls von mir in der Masius'schen Rundschau Folge, Jahrgang 1890, S. 114 und ff.) beschrieben ist. Allein auch diese Methode...

Source: Karup [1893, p.37].

The difference of two of these logarithms, $\log L_{(x+\Delta)} - \log L_{(x)}$, is, as we have seen, the integral of $\phi'_{(x)} dx$ in the limits x and $x + \Delta$ taken inversely; and in this value, better in $\phi'_{(x)}$, we must look for the qualities so well characterized by Mr. Edmonds; but it ought to be borne in mind that we cannot expect to find anything but numerical results and numerical laws.

I must beg your indulgence for having trespassed so largely on your space,

And remain,

Sir,

Yours most obediently,

*Hamburg, 15th August, 1861.**

WILHELM LAZARUS.

* We are enabled at length to find room for this letter, referred to in our "Notice to Correspondents," October, 1861,—ED. A. M.

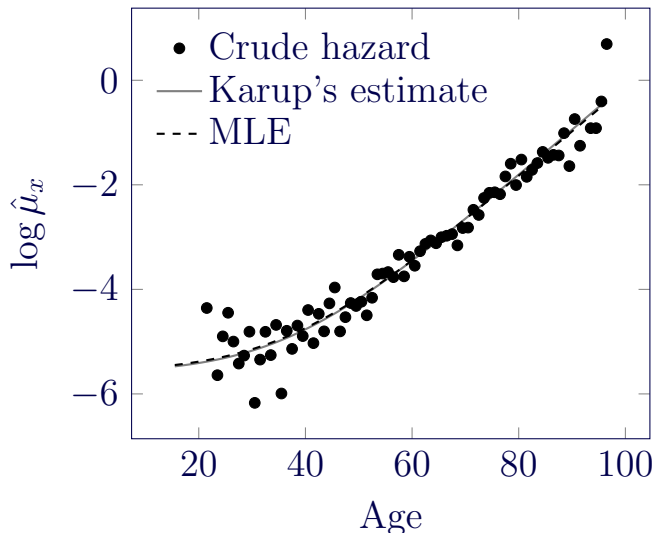
- Born in Copenhagen.
- Worked as an actuary in the German Empire.
- Karup [1893] valued the Gotha civil service pension scheme using a Makeham model for mortality fitted to the scheme's own experience.

Compare parameter estimates using:

- (a) Methodology of King [1887], and
- (b) Maximum-likelihood estimation.

Methodology	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\epsilon}$
(a) Karup [1893, p.38]	-8.81	0.0873	-5.62
(b) Maximum likelihood	-8.73	0.0861	-5.60

Source: Richards [2026, Table 1].



Source: Richards [2026, Figure 2].

- The “most frequently cited statistics paper” [van Noorden et al., 2014, p.552].
- Of considerable use to actuaries [Richards and Macdonald, 2025].
- Core result discovered by a German actuary.

It was proposed as early as 1912 by Böhmer [6] (referred to by Seal [26]), but seems to have been lost sight of by later writers and not further investigated.

Kaplan and Meier [1958, p.461]

- German actuary.
- Academic who also occasionally worked for the insurance regulator[†].
- Proposed product-limit estimator 46 years before Kaplan and Meier [1958].

[†] DGVFM [1957, p.134].

Product-limit estimators for survival probability:

$$\hat{P}(t) = \prod_{j=1}^k (n_j'/n_j) \quad \text{Kaplan and Meier [1958, eq.2b]}$$

$$1 - \gamma_h = \prod_h \frac{A_n}{A_{n-1}} \quad \text{Böhrer [1912, eq.4]}$$

- Böhmer's work is directly relevant to actuaries[†].
- So why didn't the profession take it forward?
- One answer lies with calculation technology...

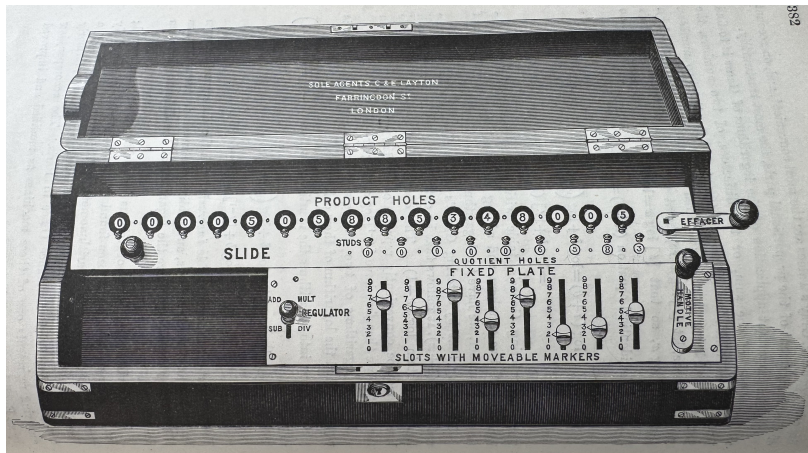
[†] Numerous practical examples are given in Richards and Macdonald [2025, Appendix A].

- Gray [1849] contains tables for life contingencies.
- The author performed 9,200 manual calculations.
- The preface thanks four colleagues for performing thousands of other manual calculations each.
- Later actuaries had mechanical assistance...

“Calculating machines of various kinds have been constructed; but the only one which has been found practically useful in the daily work of the actuary is the arithmometer.”

King [1887, p.381]

Arithmometer with four arithmetical operations.



Source: King [1887, p.382]



“initial experience with the arithmometer was not particularly good. Records within the Prudential show that the machines frequently broke down”

Campbell-Kelly [1992, p.127]

The Brunsviga mechanical calculator was presented to a meeting of German engineers in Hanover in 1892. It still had just the same four arithmetical operations.



“The Brunsviga 13 RK had 985 parts, of which 456 were different.”

Faulstich [1992, p.109]

The 1952 price list for Brunsviga calculating machines, with higher prices for variants supporting more significant digits. For comparison, the average annual salary of an employee in West Germany was 1,956 marks[†].

Modell		Preis	Stellenzahl
Brunsviga 11M	(110 V)	“ 820,-	7 x 6 x 11
	(220 V)		7 x 6 x 11
Brunsviga 11 E	(110 V)	“ 860,-	7 x 6 x 11
	(220 V)		7 x 6 x 11
Brunsviga 13 Z		“ 595.-	10 x 8 x 13
Brunsviga 13 RK		“ 795.-	10 x 8 x 13
Brunsviga 18 RK		“ 900.-	10 x 8 x 18
Brunsviga 20		“ 1175.-	12 x 11 x 20
Brunsviga Doppel 13 R/1		“ 1925.-	(10 + 10) x 8 x (13 + 13)
Brunsviga Doppel 13 R/2		“ 2075.-	(10 + 10) x (8 + 8) x (13 + 13)
Brunsviga Doppel 18 R		“ 2375.-	(10 + 10) x (8 + 8) x (18 + 18)

Source: Faulstich [1992, p.109]

[†] *Statistisches Bundesamt*, monthly average of 163 marks in 1952 (males and females combined).

1. Actuaries were the earliest advocates of survival models.
2. The basic unit of early calculations was the integrated hazard, $H_x(t) = \int_0^t \mu_{x+s} ds$.
3. The motivation was computational efficiency.
4. Technology severely limited what calculations actuaries could do...
... which probably blocked them from further developing survival models.

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