Seminar on stochastic projection models
Tower 42, Old Broad Street, London
Tuesday, 18th May 2010

Force of mortality v. mortality rate

Slide 4. Engineers call the same thing the *failure rate* when applying it to machines or components.

Slide 4. Some people find μ_x conceptually tricky. It can help to think of μ_x as simply being related to the daily rate of mortality, i.e. $\mu_x \approx 365 \times {}_{\frac{1}{365}}q_x$.

Slide 6. $\displaystyle\int_x^{x+t} \mu_s ds$ is known as the *integrated hazard function*. For more details, see Richards (2010).

Slide 10. E_x is the *initial exposed-to-risk*.

Slide 11. E_x^c is the *central exposed-to-risk*, also known to statisticians as the *waiting time*.

Projecting rates v. improvements

Slide 2. $\mathrm{E}[\hat{q}_x] = q_x$ and $\mathrm{Var}[\hat{q}_x] = \dfrac{q_x\,(1-q_x)}{E_x}$.

Model risk and basis risk

Slide 6. One way of controlling basis risk is to use a piggyback model.

Stochastic v. deterministic projections

Slide 1. Some actuaries disagree that the Solvency II rules demand stochastic projections. However, even if a deterministic projection model is used in valuation, it is necessary to benchmark it against one or more stochastic projection models to assess its likelihood.

Challenges with cause-of-death data

Slide 2. The mortality rates here are per 100,000 of population. Note that the publically available cause-of-death data is only available in five-year age bands, e.g. 70–74. Of particular concern to actuaries interested in longevity risk is that there is a single category for lives aged 85 and over. Data granularity is less than ideal for pensioner mortality.

Slide 3. Ten different classification systems have been in use in England and Wales over the past century, culminating with the use of ICD-10 since 2001.

Slide 4. We must also take note of uncertainty surrounding the classification of cause of death. Perhaps as a gentle reminder of this, the six causes of death listed contain one occurrence of the phrase “ill-defined” and three uses of the word “unspecified”.

Slide 6. Projecting a time series is tricky enough, as we shall see with Iain Currie’s session on ARIMA and drift models. Projecting correlated time series is particularly fraught.

Slide 7. The graph shows how prostate-cancer mortality rates have risen against a backdrop of falling mortality rates for the same group. The discontinuity in 1983 suggests a change in classification similar to that described in slide 5, even though the same ICD-9 system was used throughout the 1979–2000 period.

Slide 8. Mortality rates for the three causes of death shown are strongly linked to deprivation index, which is a measure of socio-economic status. Other measures are socio-economic group (dervied from occupation) and lifestyle (derived from geodemographic type). The use of geodemographic type is decribed in detail by Richards (2008)

Slide 9. The relationship between socio-economic status and mortality by cause is anything but simple. This matters because around half of all pensions are paid to the highest-status lives in a portfolio (Richards, 2008).

Sample paths

Slide 2. A rough sample path might be more realistic looking, but in practice it is trend risk which mainly drives the extra risk capital for an annuity or pension. This is just as adequately measured by a penalty-based projection as a time series.

Slide 2. A cautionary note on time series — some time-series confidence intervals are derived from simulation only. This often assumes the time-series parameters are known, whereas in practice these parameters have considerable uncertainty. Time-series confidence intervals without parameter uncertainty are *under-estimates* of risk.

Run-off simulations

Slide 3. This approach is of course impossible using model points. The individual contribution to run-off risk can only be assessed using full-portfolio simulation.

References

RICHARDS, S. J. **2008** *Applying survival models to pensioner mortality data*, British Actuarial Journal **14** (to appear).

RICHARDS, S. J. **2010** *A handbook of parametric survival models for actuarial use*, Longevitas Technical Paper.

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Longevitas Ltd,
16th May 2010.