

2D P-spline models

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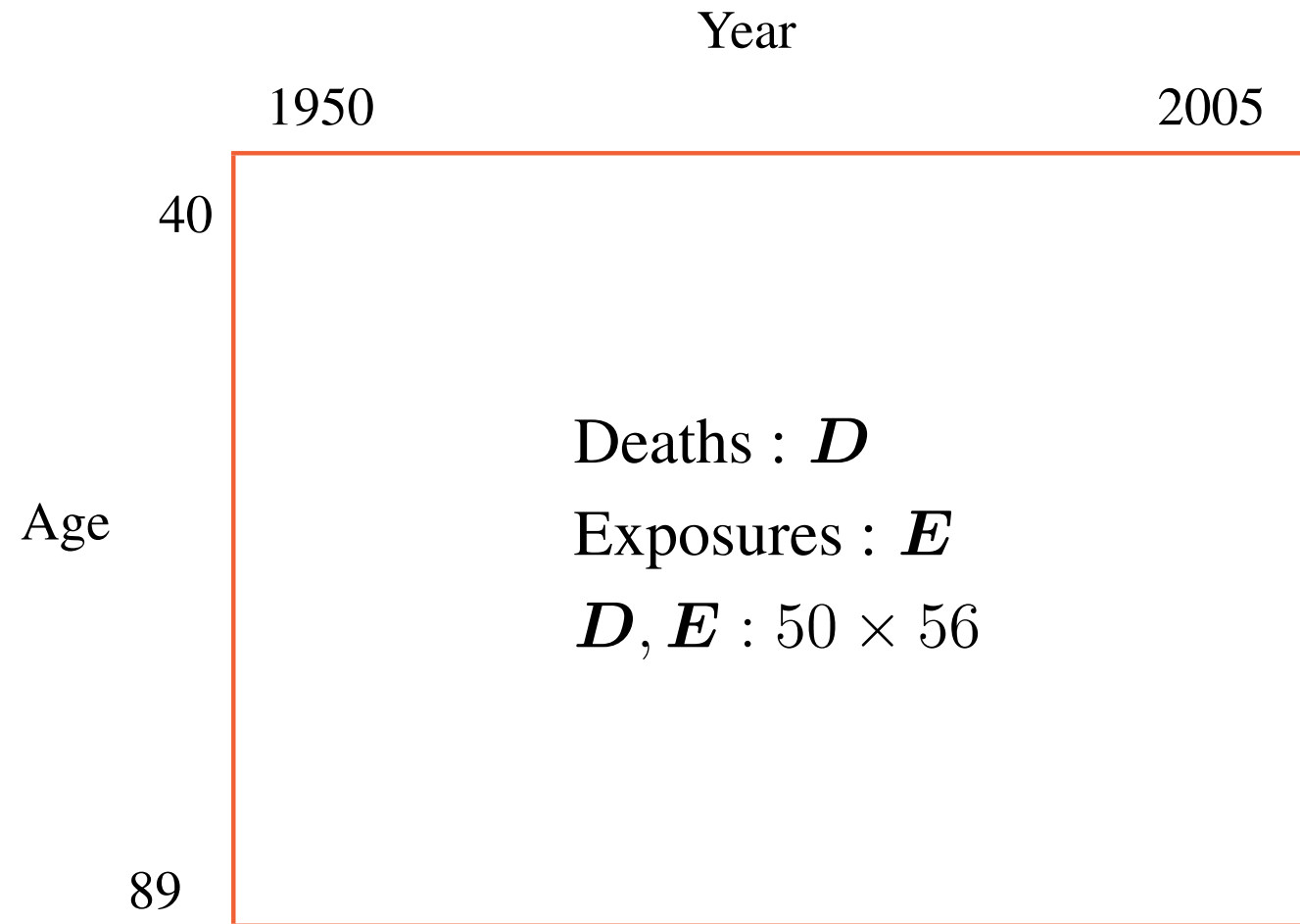
Heriot-Watt University

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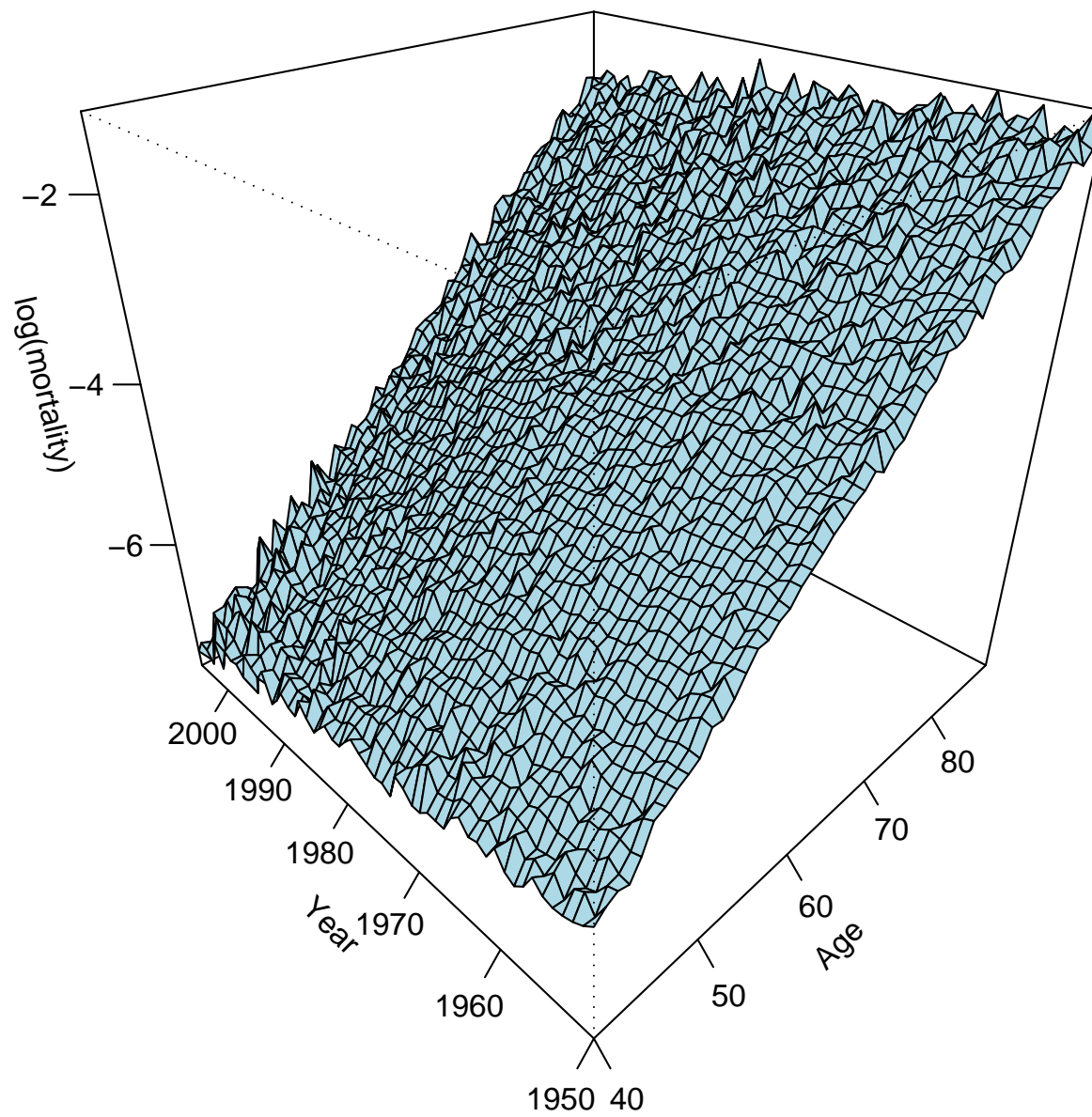


London
18th May 2010

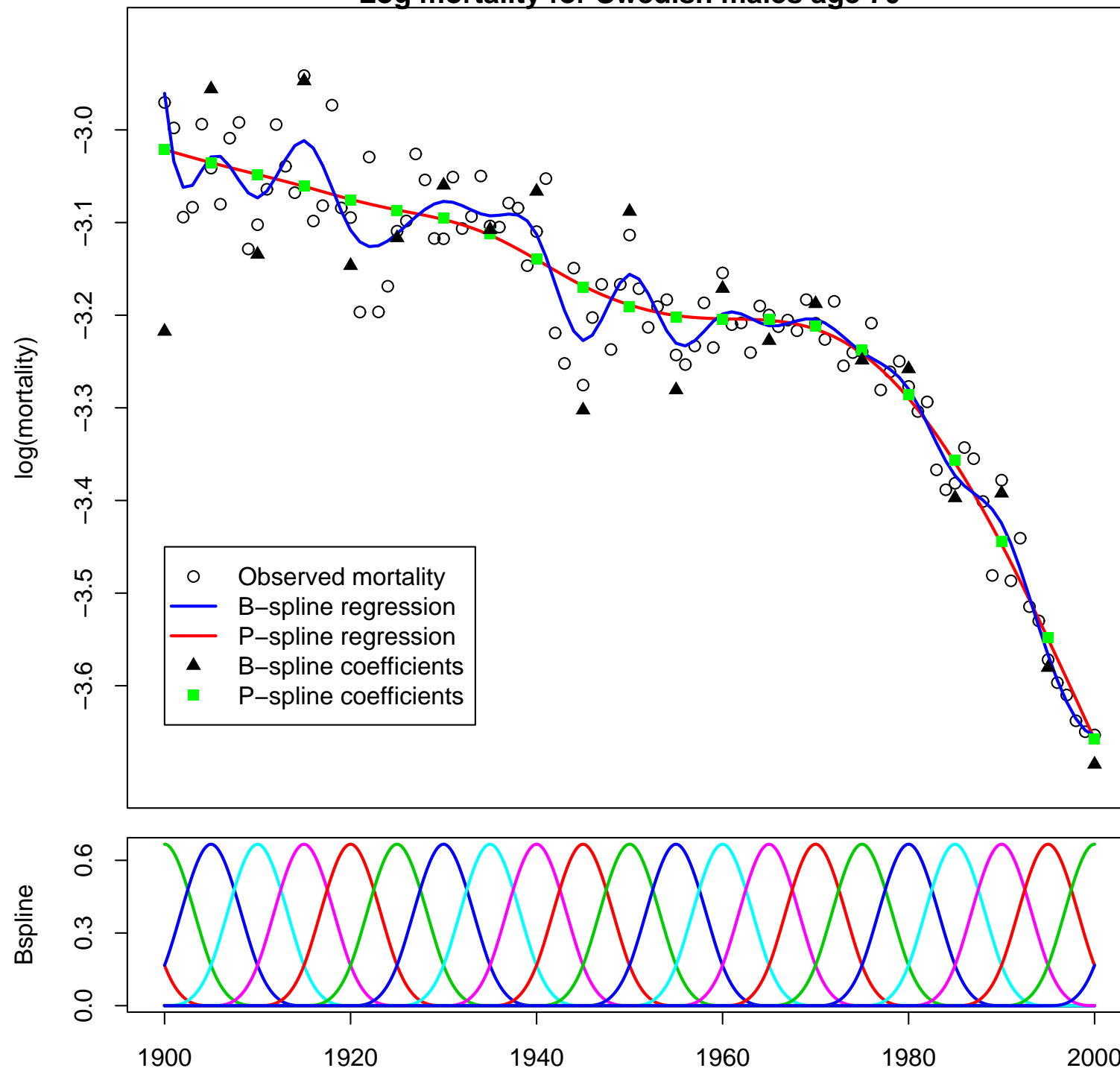
CMI claim incidence data



Raw mortality surface



Log mortality for Swedish males age 70



Penalties & Forecasting

- **Penalties** are placed on **differences** between adjacent coefficients.

$$P_2(\boldsymbol{\theta}) = (\theta_1 - 2\theta_2 + \theta_3)^2 + \dots + (\theta_{c-2} - 2\theta_{c-1} + \theta_c)^2.$$

This is a **second order roughness penalty**.

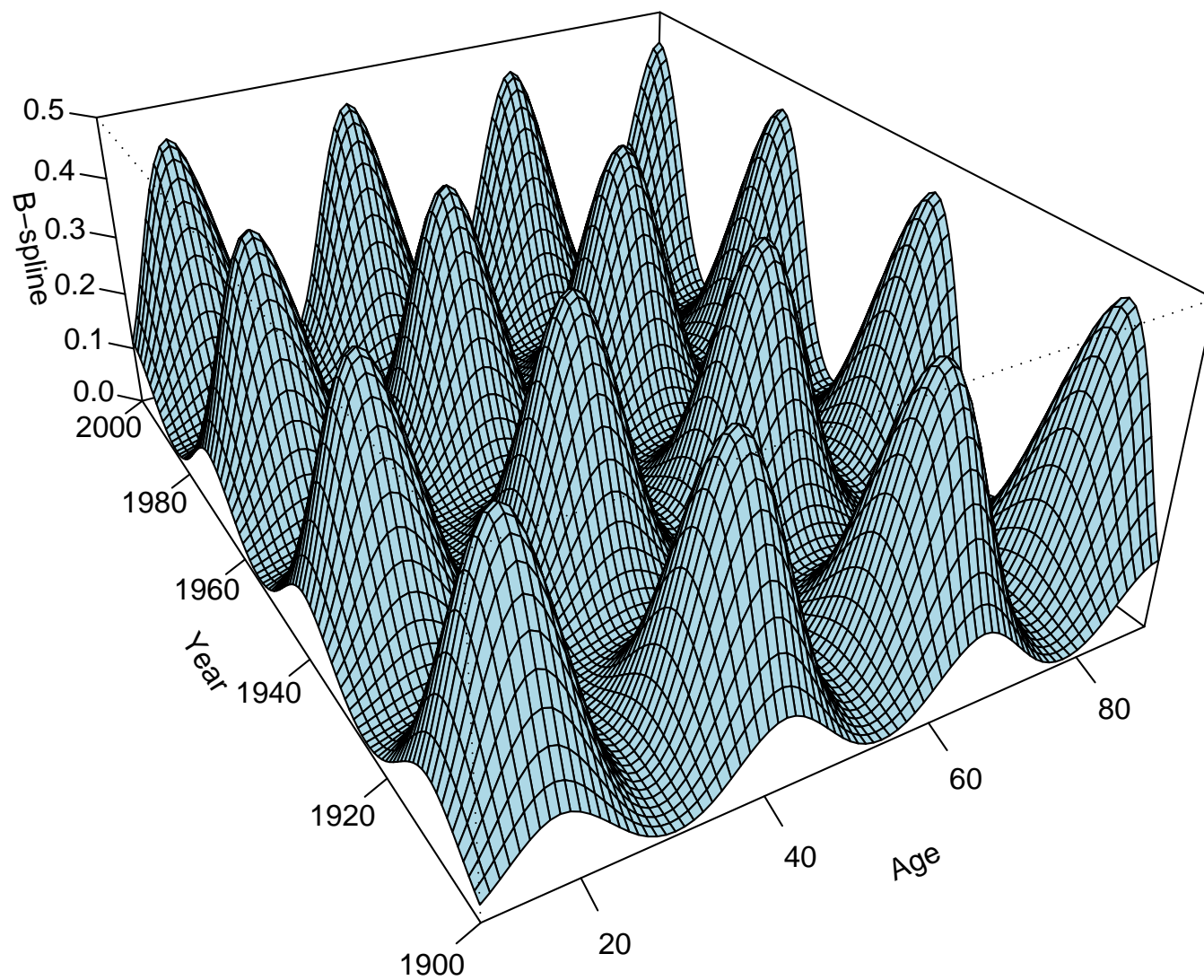
- Smoothness is a balance between **goodness-of-fit** and **roughness**. This balance is determined by the **smoothing parameter** (BIC).
- The penalty forecast is a **linear forecast** of future coefficients.

Modelling mortality tables

2-dimensional smoothing - AP

- Suppose we have two B-spline bases, one defined along age and one along year.
- **Question:** Can we combine these bases to give a 2-dimensional B-spline basis for the age-period table?
- Penalties are applied down columns (age) and along rows (period).

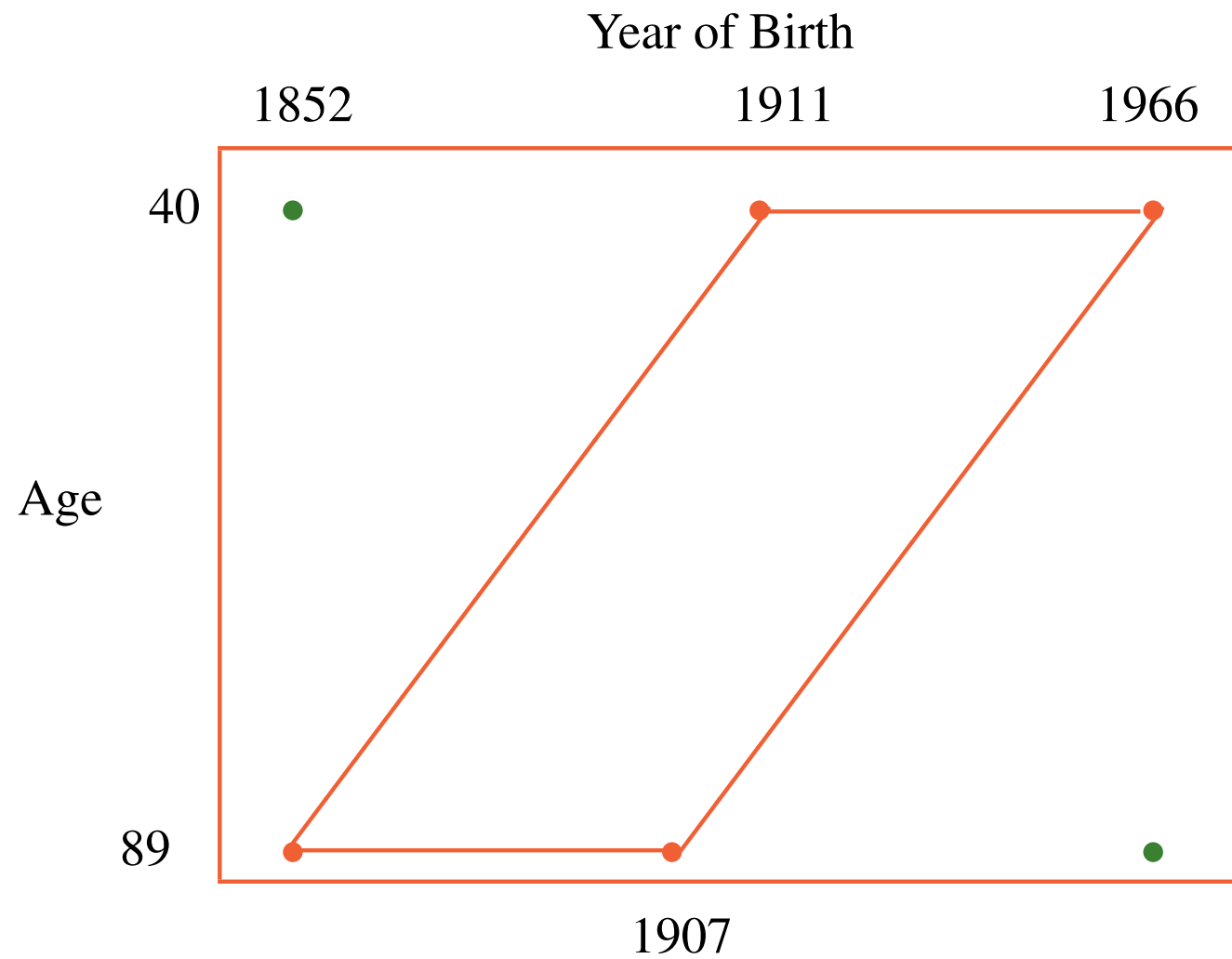
2d B-spline basis



Age-Cohort Tables I

| | | Year of Death | |
|-----|----|----------------|------|
| | | 1950 | 2005 |
| Age | 40 | 1911 | 1966 |
| | 89 | 1852 | 1907 |
| | | Year of Birth | |
| | | 50×56 | |

Age-Cohort Tables II



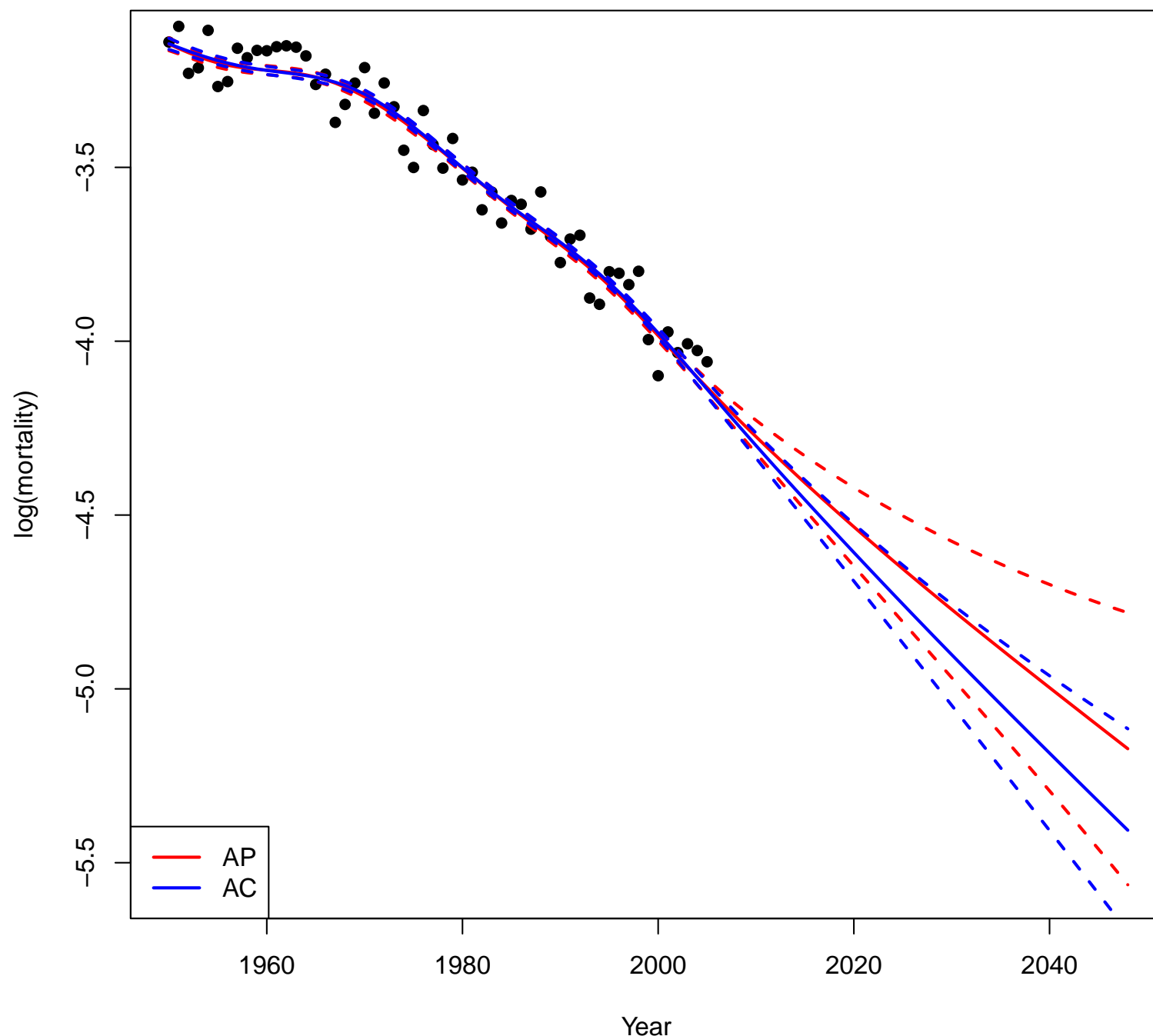
2-dimensional smoothing - AC

Column (age) and row (cohort) penalties give the **2D-AC** model and forecasts.

Example: CMI data

| | AP | AC |
|------------------------|------|------|
| BIC | 5360 | 5337 |
| λ_a | 218 | 490 |
| $\lambda_y(\lambda_c)$ | 683 | 1087 |
| DF | 45 | 37 |

AP and AC forecasts for age 70



References

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- Richards, Kirkby and Currie (2006) British Actuarial Journal, 12, 5-61.
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and papers in
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