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Force of mortality or mortality rate?

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Mortality rates

- A mortality rate is the proportion of lives dying aged x, denoted q_x
- $q_x \in [0, 1]$

Mortality rates

- A more rigorous definition is as a probability of death:

$$q_x = \Pr(\text{life aged } x \text{ dies before age } x + 1 | \text{alive at age } x)$$

Mortality rates over period h

- A mortality rate over a period of time h is denoted ${}_hq_x$

$$ {}_hq_x = \Pr(\text{life aged } x \text{ dies before age } x+h | \text{alive at age } x) $$

- q_x is shorthand for ${}_1q_x$

Mortality hazard

- The mortality hazard is the continuous-time equivalent of q_x
- Usually denoted μ_x
- Actuaries call this the *force of mortality*:

$$\mu_x = \lim_{h \to 0^+} \frac{{}_hq_x}{h}$$

Advantages of μ_x over q_x

- Simpler limits: $q_x \in [0, 1]$ but $\mu_x > 0$
- Direct and exact derivation of q_x from μ_x:

$$q_x = 1 - \exp\left(-\int_x^{x+1} \mu_s ds\right)$$

- Converse not true: μ_x can only be approximated from q_x

Advantages of μ_x over q_x

- μ_x leads to a *survival model*:

$$ {}_tp_x = \exp\left(-\int_x^{x+t} \mu_s ds\right) $$

where ${}_tp_x$ is the probability of surviving from age x to age $x+t$.

Multi-state models

- In a multi-state or Markov model, μ_x is a *transition intensity*
- Using q_x in a multi-state model requires additional (unrealistic) assumptions

Multi-state models — example

- Term assurance can either lapse or lead to claim
- Lapses are selective: only the healthy lapse!

Multi-state models — example

- Models using q_x assume independence of decrements — not true!
- Models using μ_x don't need to assume independence
- μ_x models are more realistic than those using q_x

Estimation of q_x

- d_x = number of deaths aged x
- E_x = number of lives aged exactly x at start of year
- $\hat{q}_x = \dfrac{d_x}{E_x}$

Estimation of μ_x

- d_x = number of deaths aged x
- E^c_x = time lived aged x
- $\hat{\mu}_x = \dfrac{d_x}{E^c_x}$

Approximation

- E_x^c = time lived aged x
- $E_x^c \approx E_x - 0.5d_x$

Conclusions and questions

- μ_x more usable measure than q_x
- q_x requires more (unrealistic) assumptions in multi-state models
- Simple derivation of q_x from μ_x if required

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