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Projecting rates or improvements?

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Mortality improvement rates

- Mortality improvements were introduced by Willets (1999).
- Based on mortality rate $q_{x,y}$ at age x in year y.
- A *mortality improvement* $i_{x,y}$ is the relative change in mortality rates $q_{x,y}$ and $q_{x,y-1}$:

$$i_{x,y} = 1 - \frac{q_{x,y}}{q_{x,y-1}}$$

- $i_{x,y} \in (-\infty, +\infty)$

Estimation and projection of $\mu_{x,y}$

- Estimation of $\mu_{x,y}$ requires $d_{x,y}$ and $E^c_{x,y}$
- $\hat{\mu}_{x,y} = \dfrac{d_{x,y}}{E^c_{x,y}}$
- $\hat{\mu}_{x,y}$ has simple properties:

$$\mathrm{E}[\hat{\mu}_x] = \mu_x$$

$$\mathrm{Var}[\hat{\mu}_x] = \frac{\mu_x}{E^c_x}$$

- $\hat{q}_{x,y}$ has similarly simple properties using $d_{x,y}$ and $E_{x,y}$.

Estimation and projection of $i_{x,y}$

- The estimator $\hat{i}_{x,y}$ is a ratio of two random variables
- The variance of $\hat{i}_{x,y}$ is a particularly messy function!

Conclusions and questions

- It is (much!) simpler to estimate and project $\mu_{x,y}$ (or $q_{x,y}$) than $i_{x,y}$

References

Willets, R. C. **1999** *Mortality in the next millennium*, Staple Inn Actuarial Society, London