Forecasting with Time Series: ARIMA and Drift Models

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Lee-Carter model

Lee-Carter (1992)

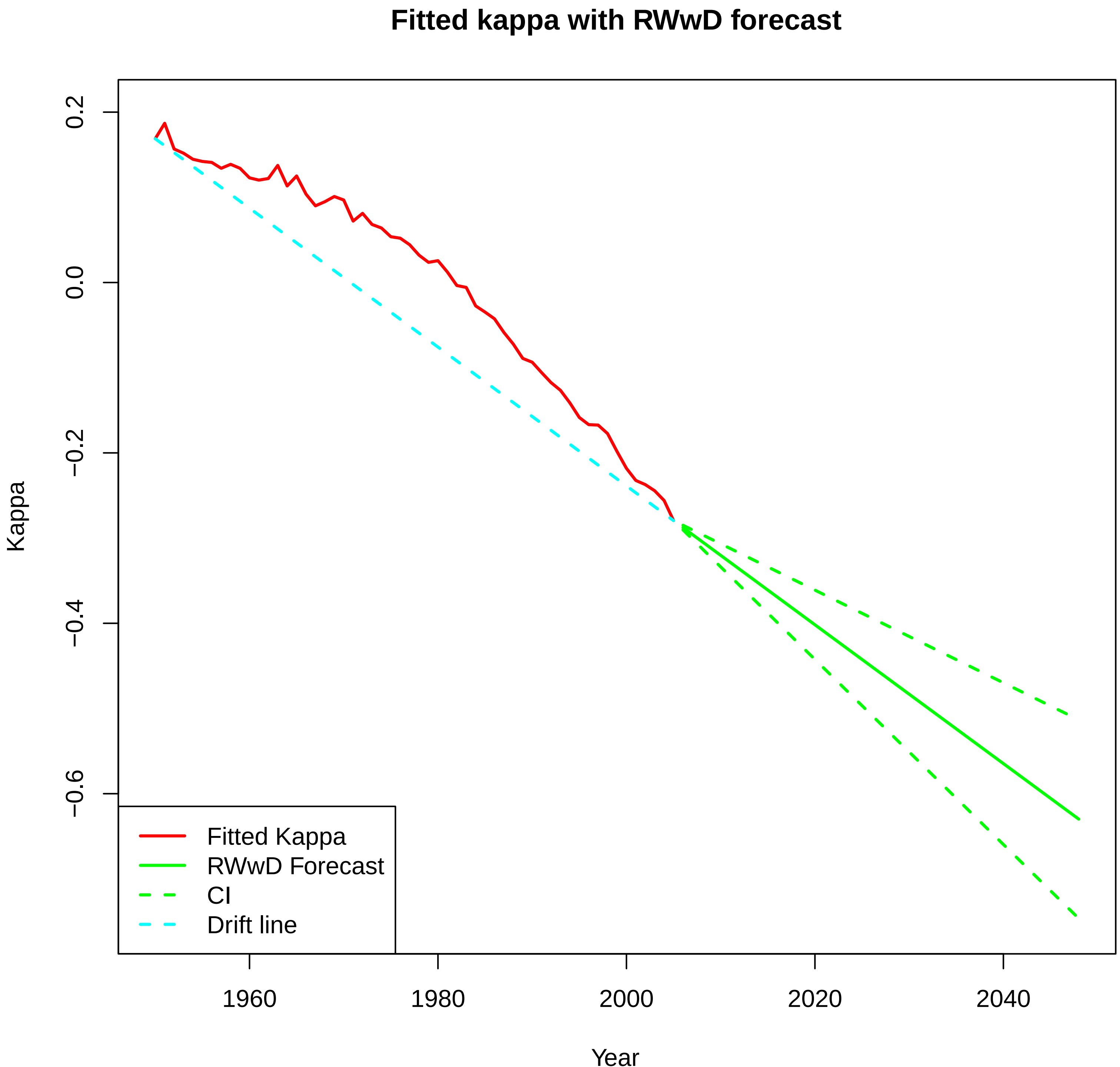
$$\log \mu_{ij} = \alpha_i + \beta_i \kappa_j, \ i = 1, \ldots, n_a, \ j = 1, \ldots, n_y = n.$$

The random walk with drift forecast (Lee-Carter, 1992):

$$\hat{\kappa}_{n+1} = \hat{\kappa}_n + \hat{a}$$

where

$$\hat{a} = \frac{1}{n-1} \sum_{1}^{n-1} (\hat{\kappa}_{i+1} - \hat{\kappa}_i) = \frac{\hat{\kappa}_n - \hat{\kappa}_1}{n-1}.$$


Fitted kappa with RWwD forecast
Kappa
0.2
0.0
−0.2
−0.4
−0.6
1960
1980
2000
2020
2040
Year
Fitted Kappa
RWwD Forecast
CI
Drift line

ARIMA models

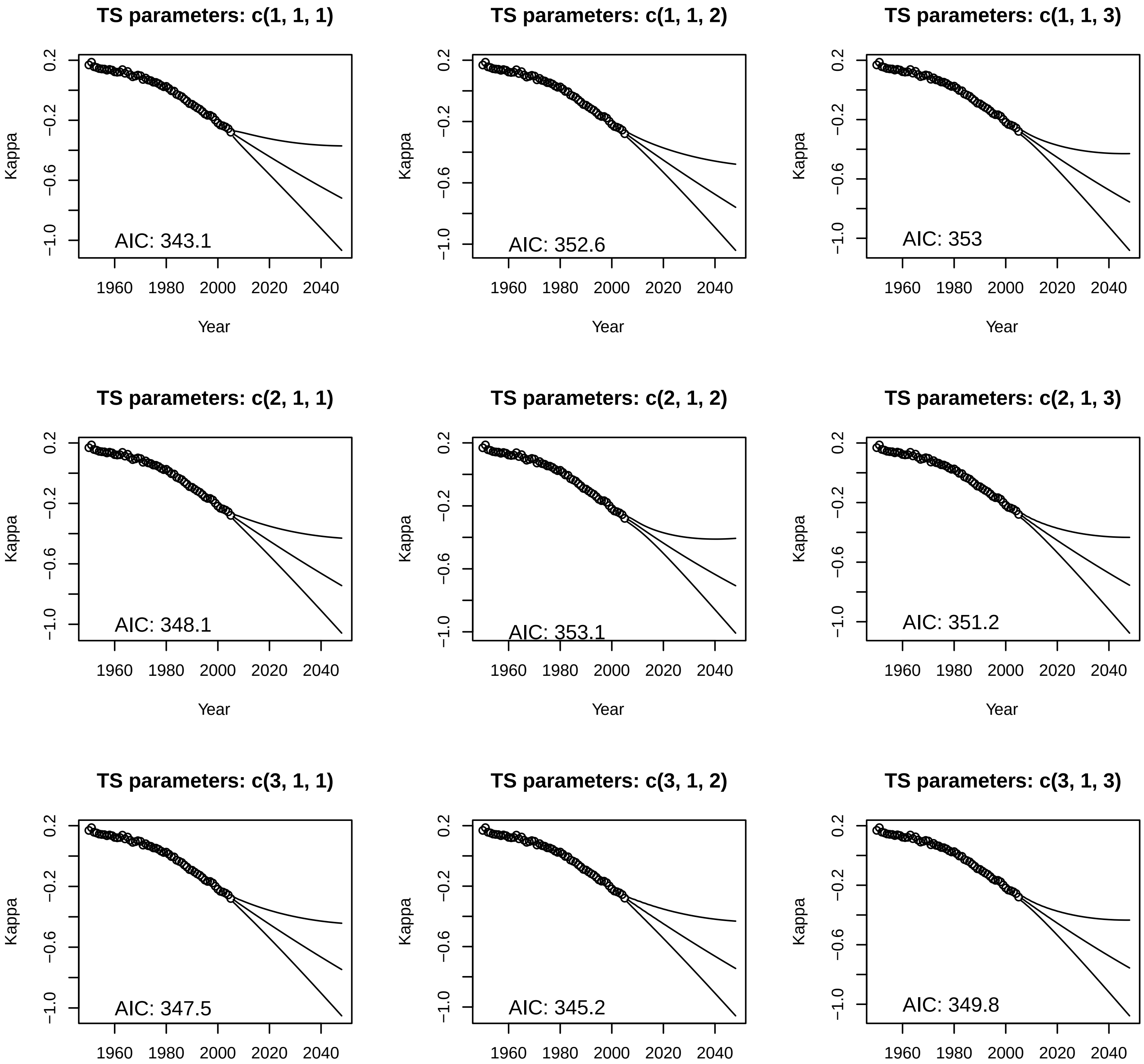
$$\text{ARIMA}(p, d, q) : \phi(B)(1 - B)^d \kappa_t = \theta(B) z_t$$

where B is the backward-shift operator.

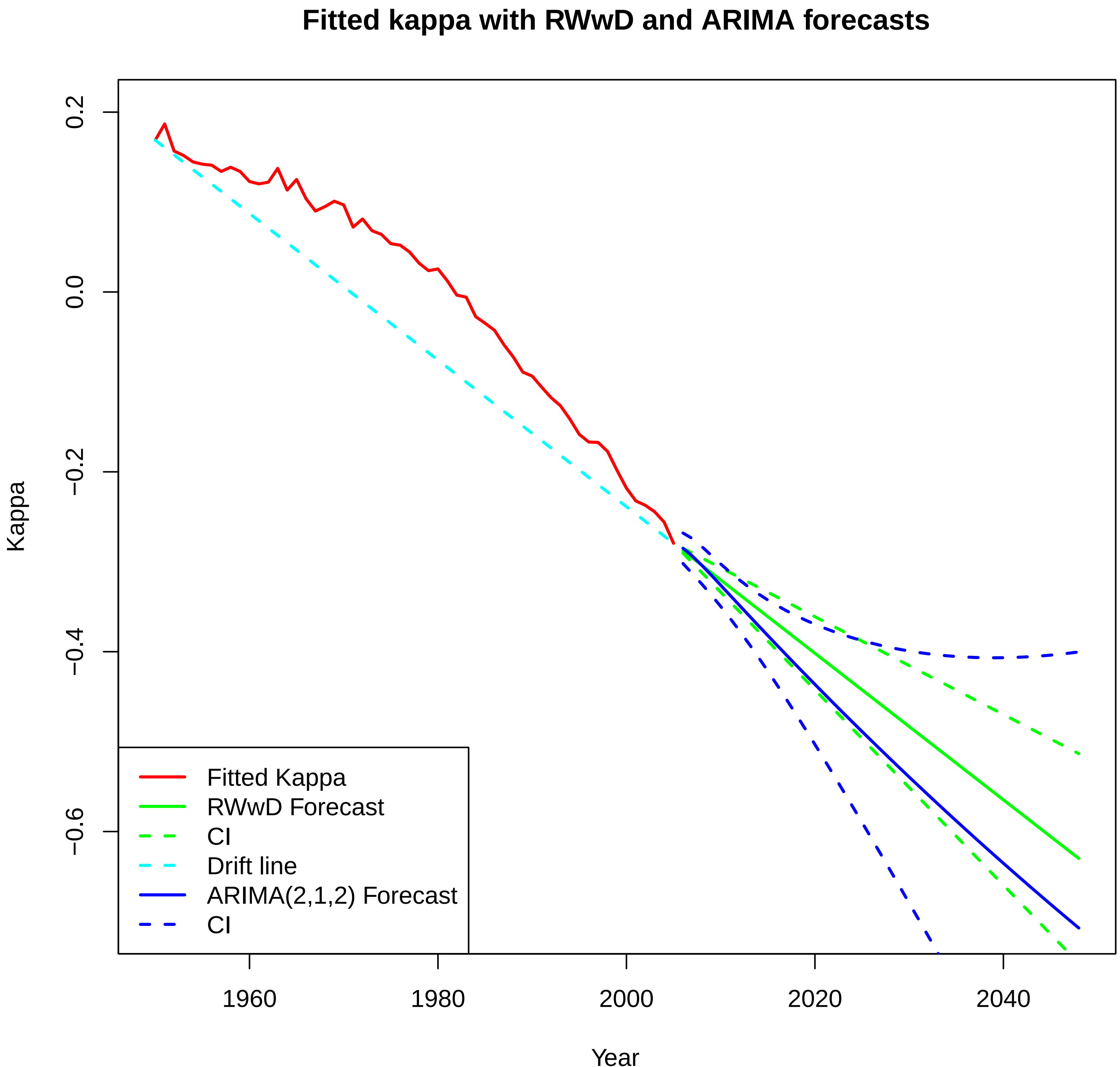
Example: $\text{ARIMA}(1, 1, 1)$ with $p = 1$, $d = 1$, $q = 1$

$$\begin{aligned}
(1 - 0.2B)(1 - B)\kappa_t &= (1 - 0.3B) z_t \\
\Rightarrow \kappa_t &= 1.2\kappa_{t-1} + 0.2\kappa_{t-2} - 0.3z_{t-1} + z_t \\
&= \kappa_{t-1} + 0.2\kappa_{t-1} + 0.2\kappa_{t-2} - 0.3z_{t-1} + z_t
\end{aligned}$$

Model selection is via Akaike Information Criterion (AIC).



TS parameters: c(1, 1, 1)
AIC: 343.1
TS parameters: c(1, 1, 2)
AIC: 352.6
TS parameters: c(1, 1, 3)
AIC: 353
TS parameters: c(2, 1, 1)
AIC: 348.1
TS parameters: c(2, 1, 2)
AIC: 353.1
TS parameters: c(2, 1, 3)
AIC: 351.2
TS parameters: c(3, 1, 1)
AIC: 347.5
TS parameters: c(3, 1, 2)
AIC: 345.2
TS parameters: c(3, 1, 3)
AIC: 349.8
Kappa
Year
0.2
−0.2
−0.6
−1.0
1960
1980
2000
2020
2040


Fitted kappa with RWwD and ARIMA forecasts
Kappa
0.2
0.0
−0.2
−0.4
−0.6
1960
1980
2000
2020
2040
Year
Fitted Kappa
RWwD Forecast
CI
Drift line
ARIMA(2,1,2) Forecast
CI