

The Lee-Carter Family

Iain Currie

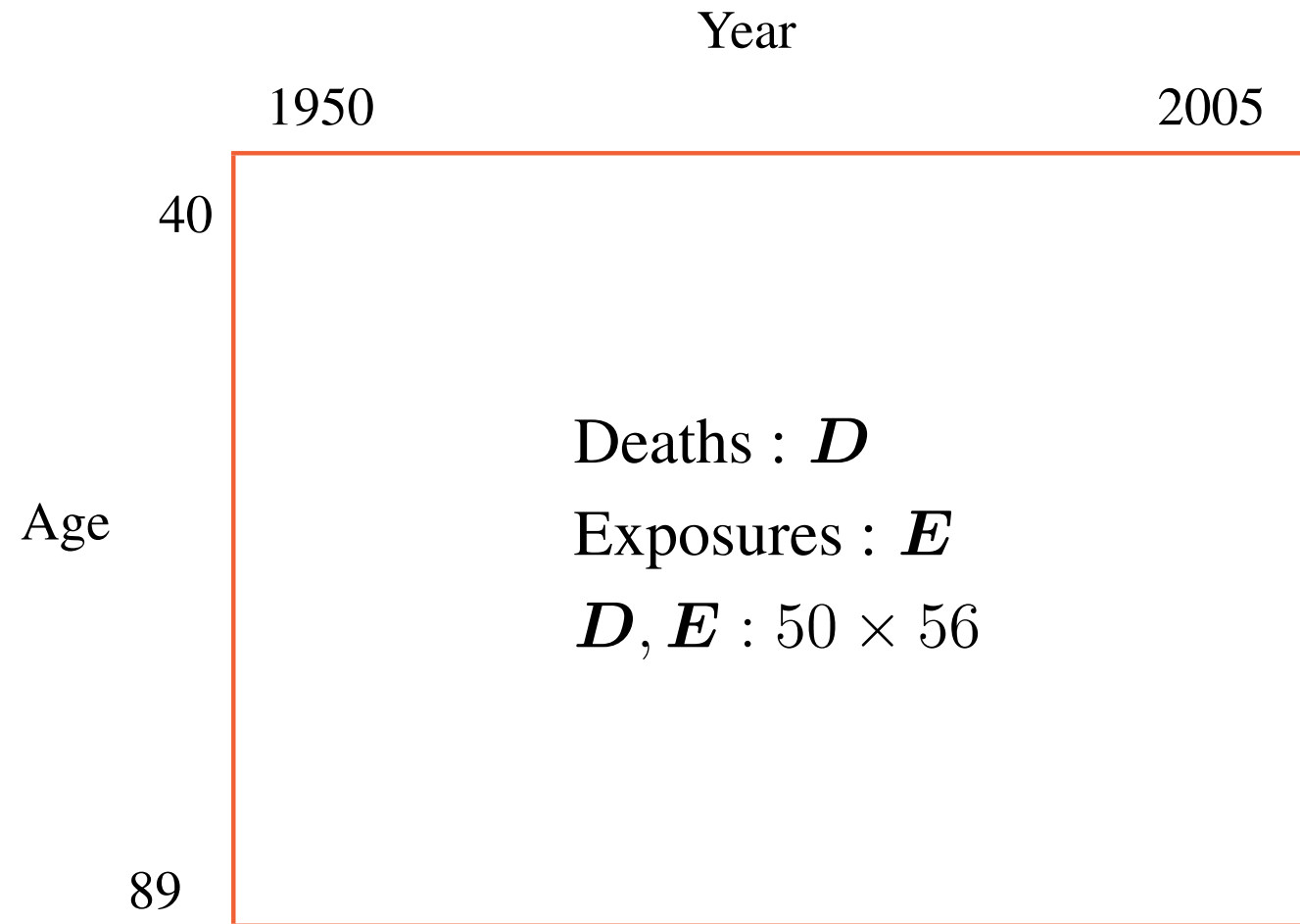
Heriot-Watt University

&

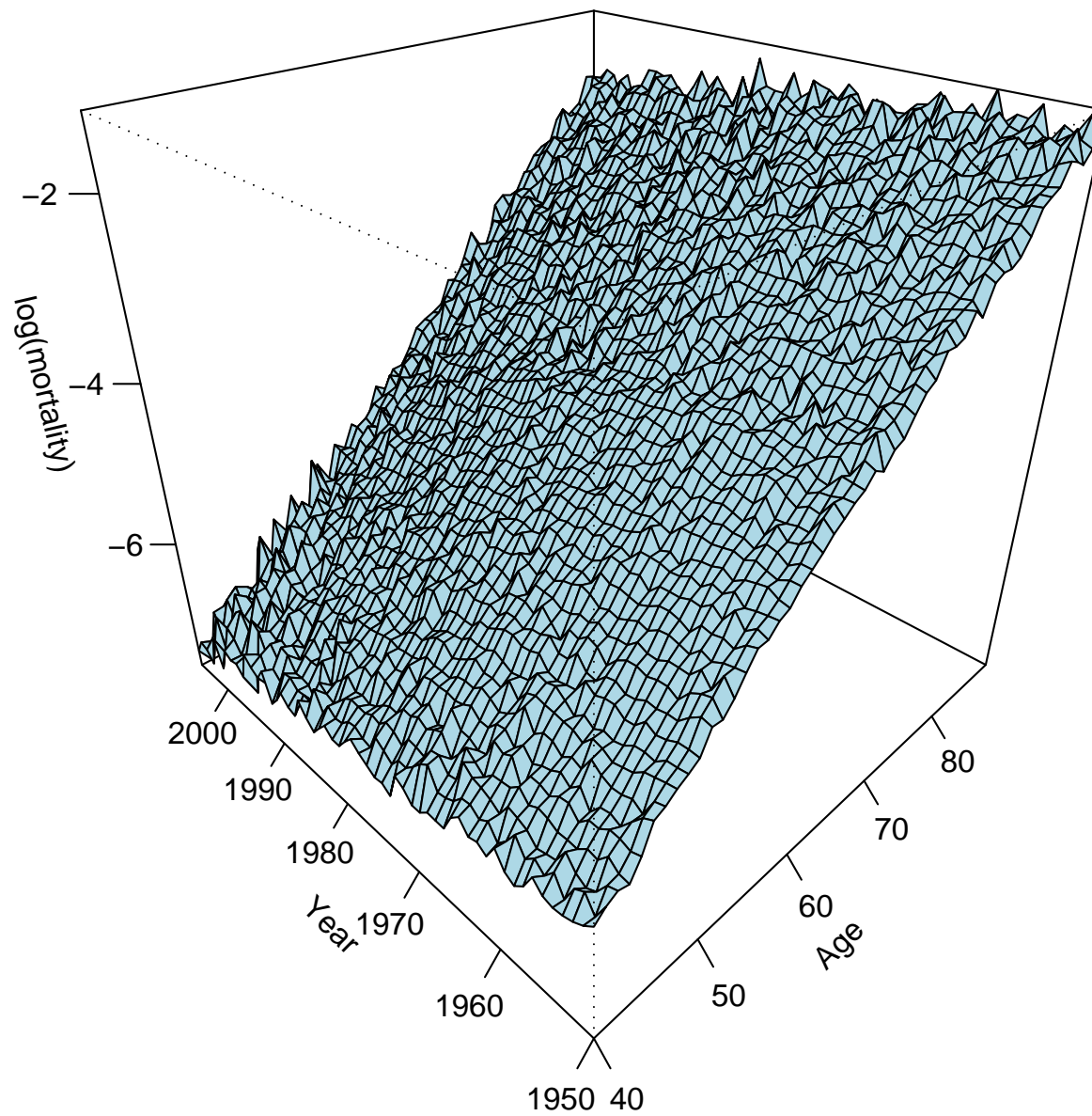


London
18th May 2010

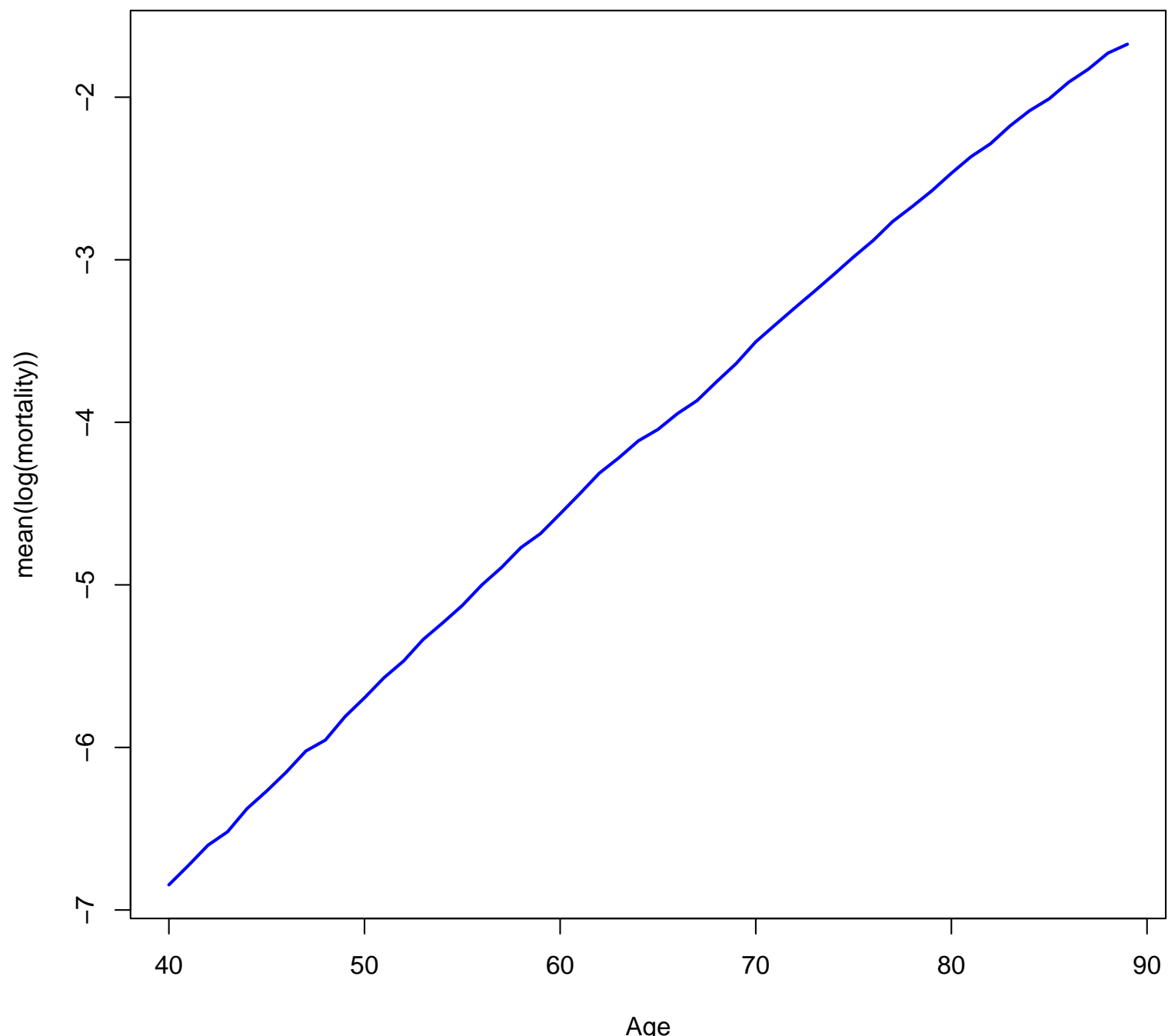
CMI claim incidence data



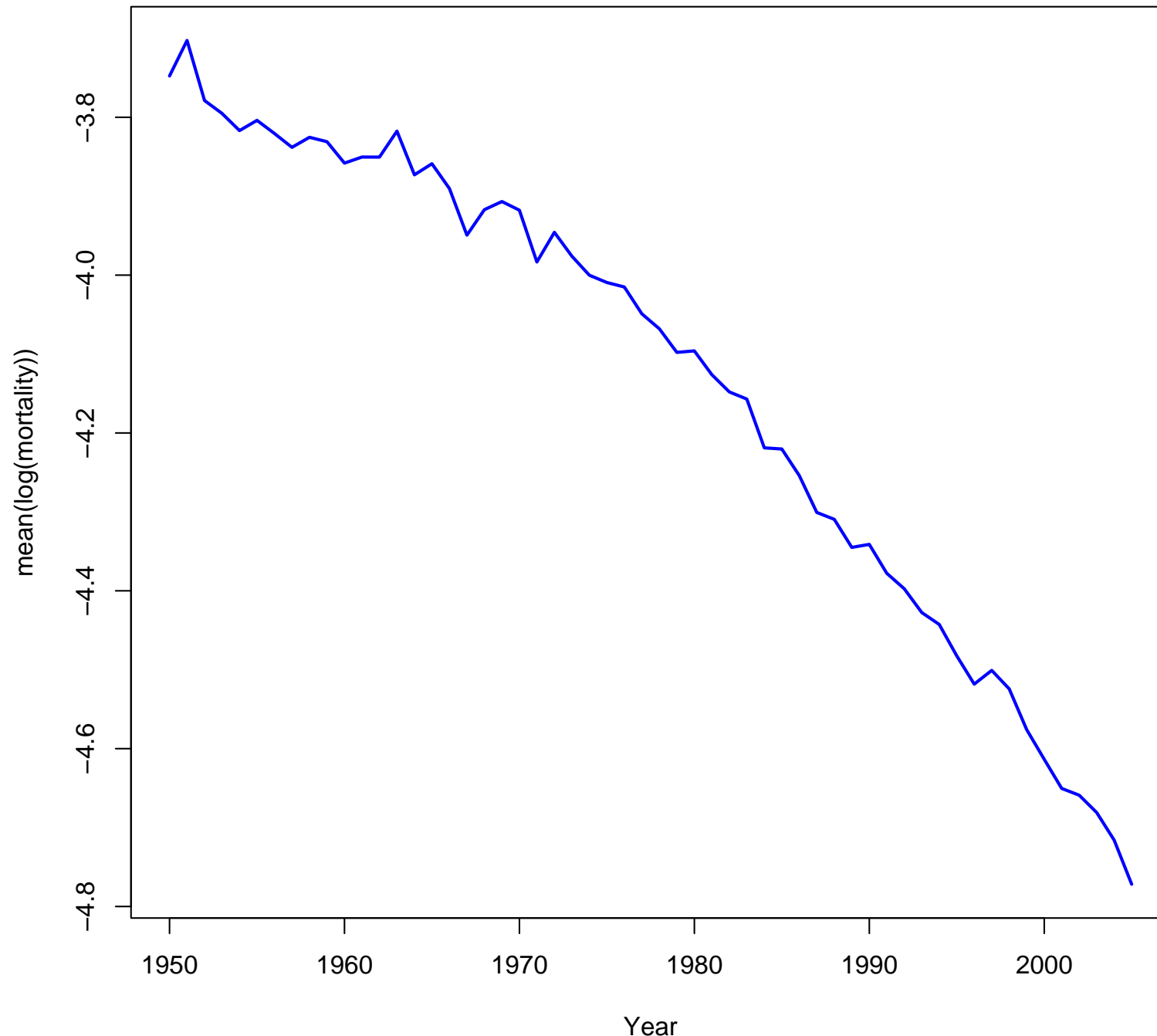
Raw mortality surface



Log(mortality) by age



Log(mortality) by year

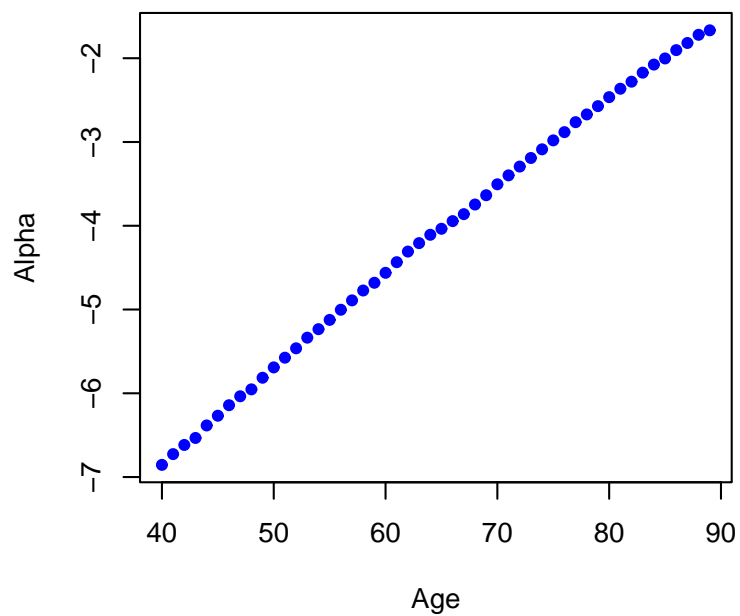


Lee-Carter models

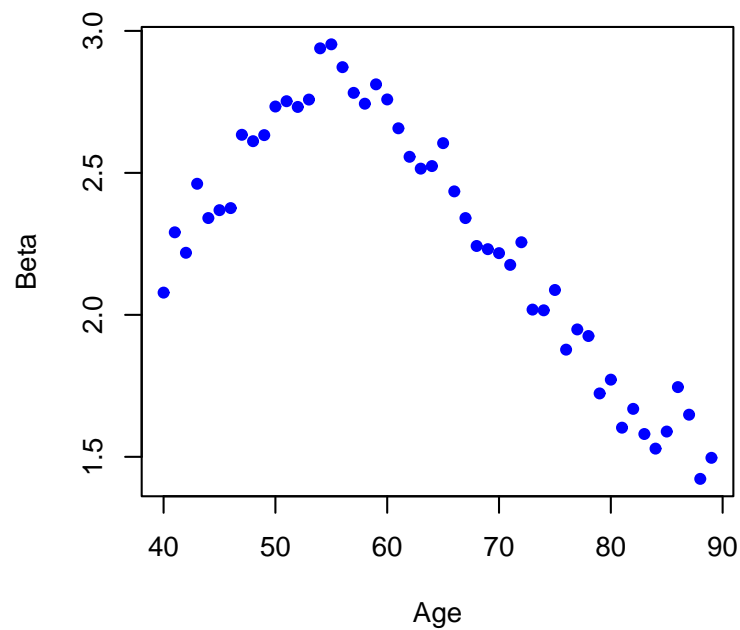
Lee-Carter (1992)

$$\begin{aligned}\log \mu_{ij} &= \alpha_i + \beta_i \kappa_j, \quad i = 1, \dots, n_a, \quad j = 1, \dots, n_y, \\ \sum \kappa_j &= 0, \quad \sum \kappa_j^2 = 1.\end{aligned}$$

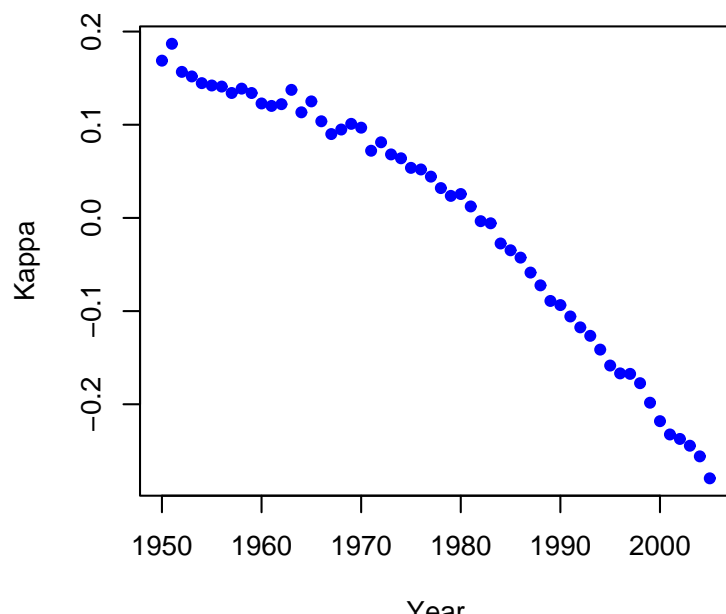
Original LC: Alpha



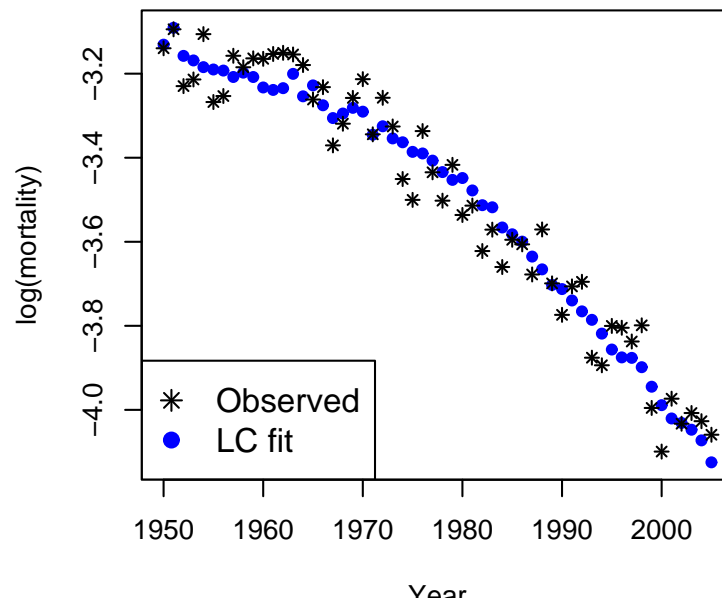
Original LC: Beta



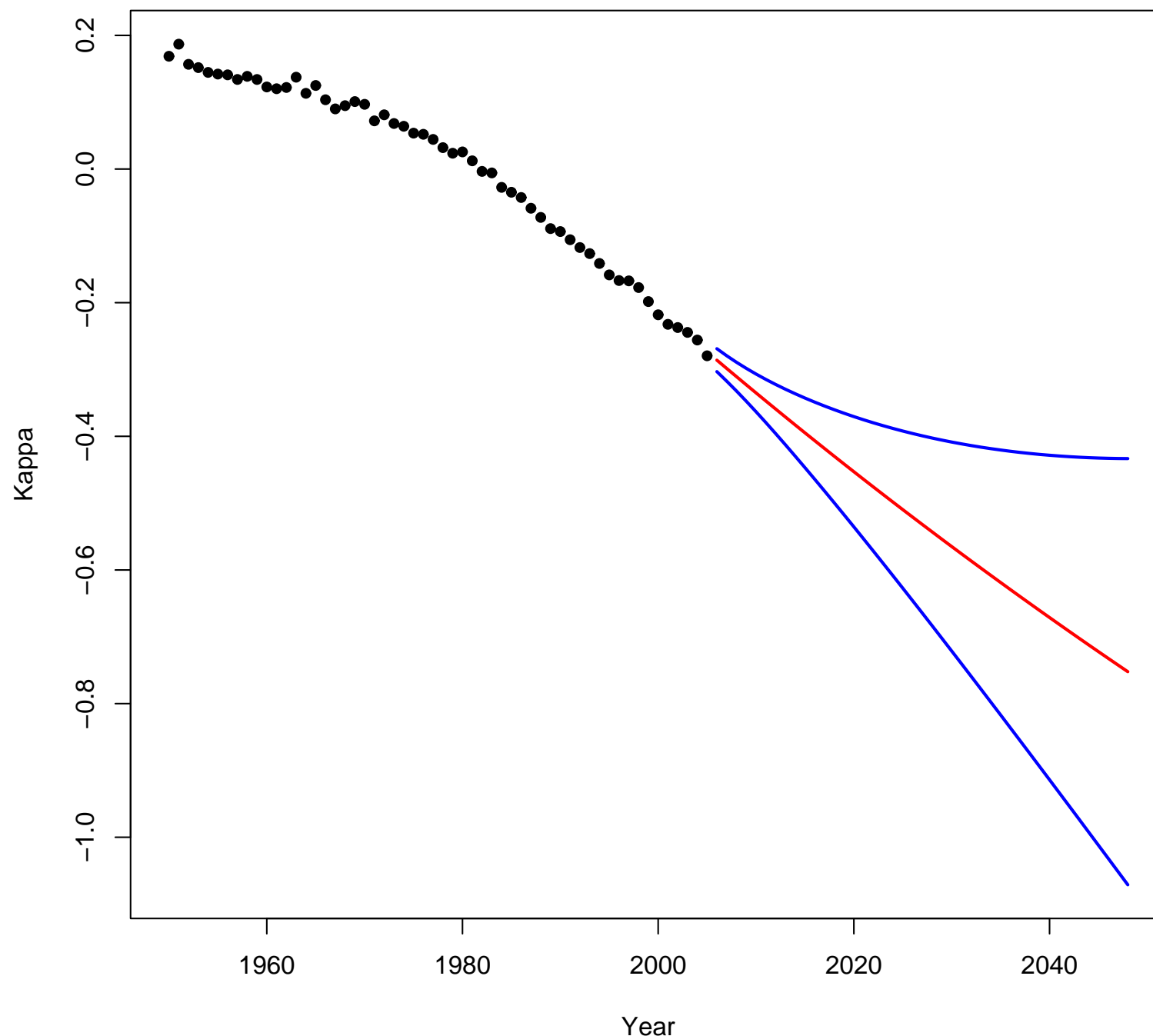
Original LC: Kappa



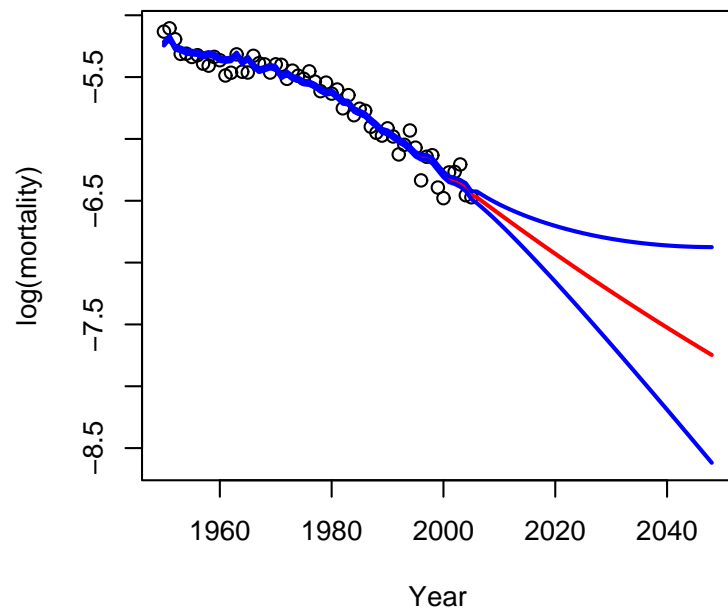
Original LC: Age 70



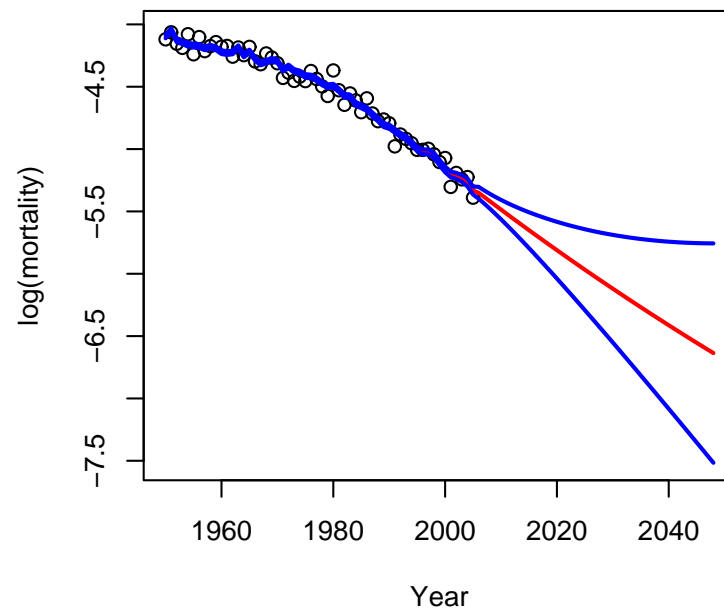
Estimated kappa with ARIMA forecast & 95% CIs



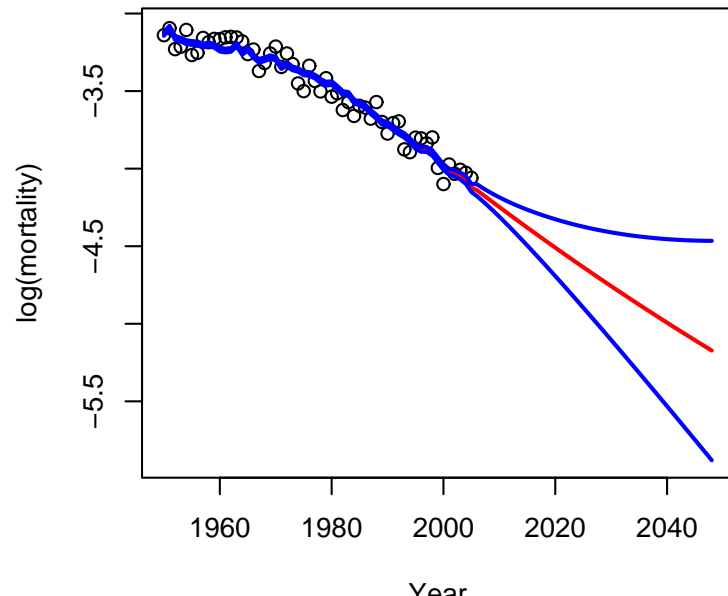
Lee-Carter forecast for age = 50



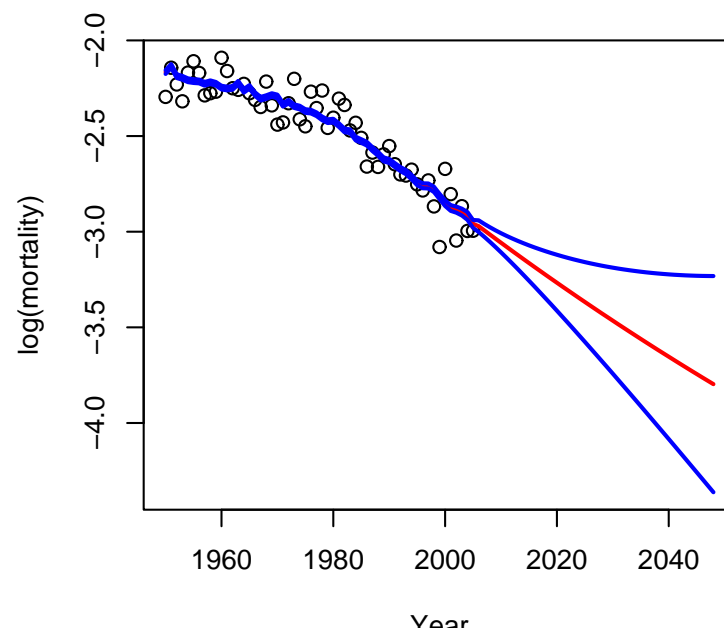
Lee-Carter forecast for age = 60



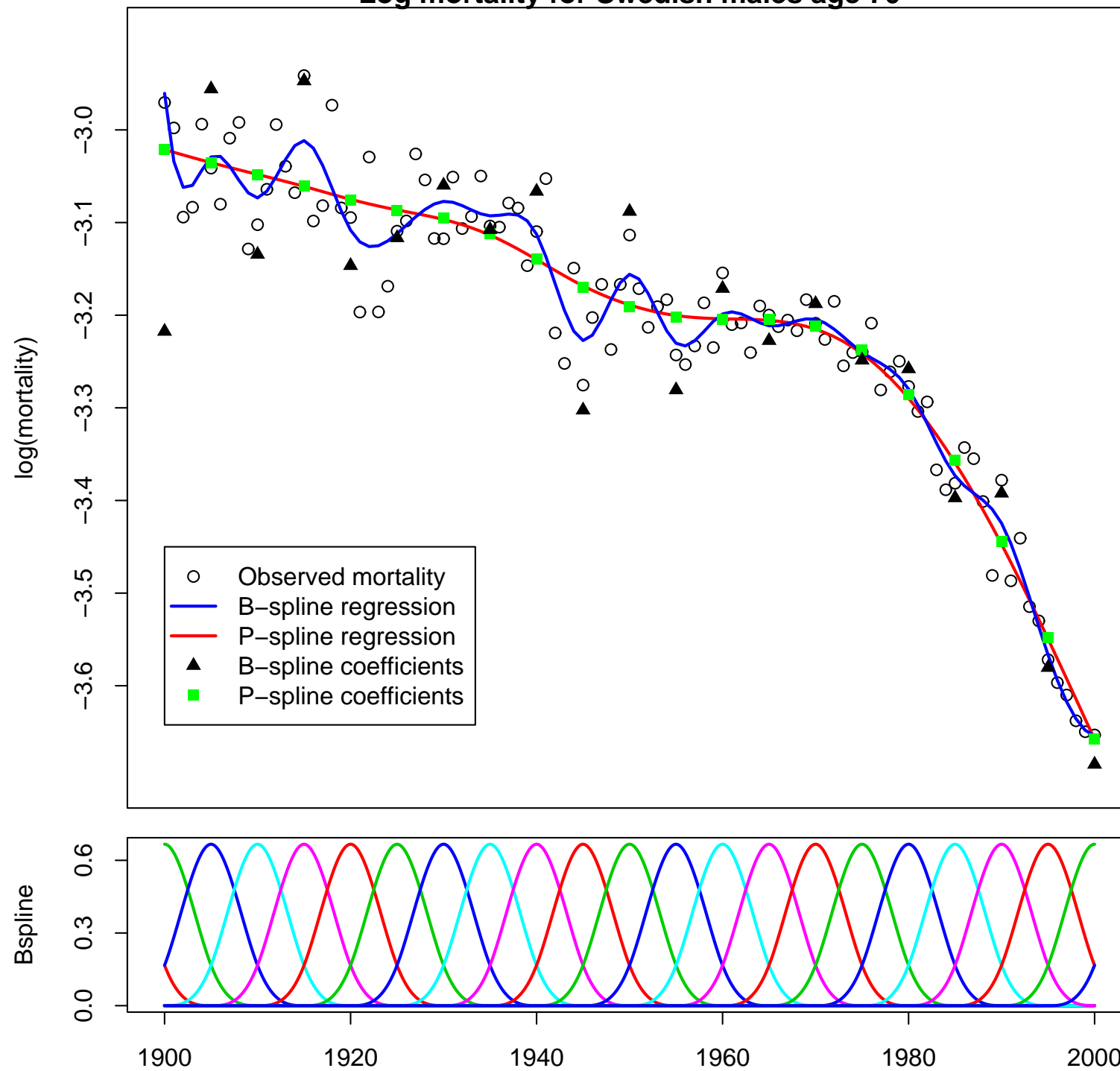
Lee-Carter forecast for age = 70



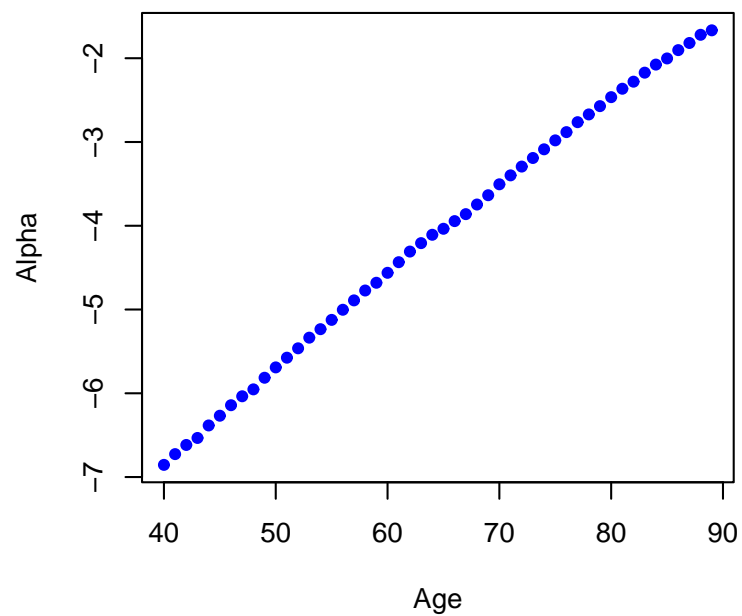
Lee-Carter forecast for age = 80



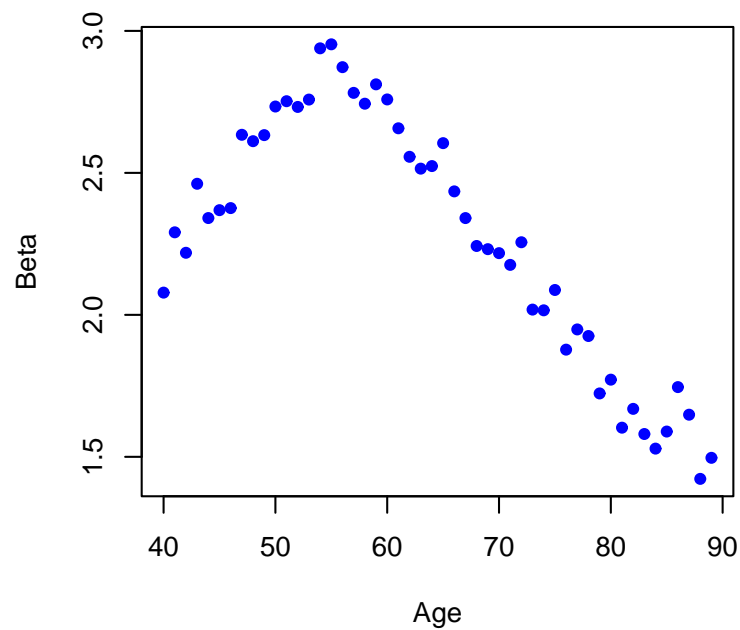
Log mortality for Swedish males age 70



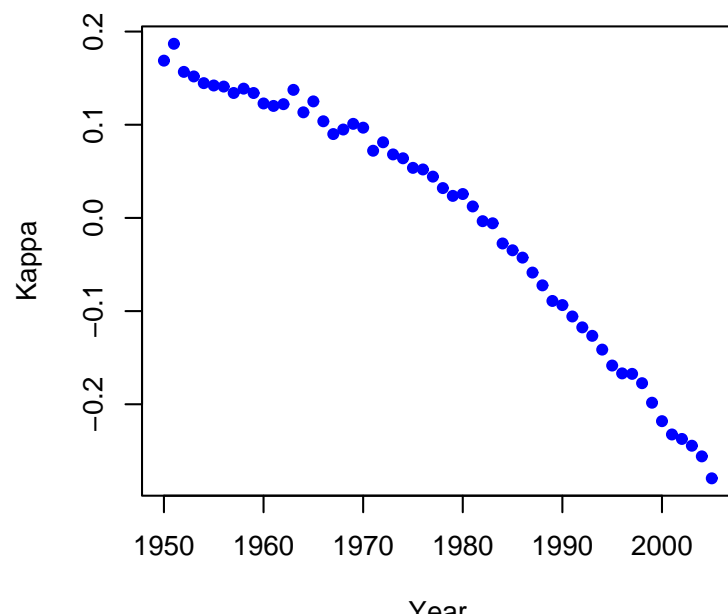
Original LC: Alpha



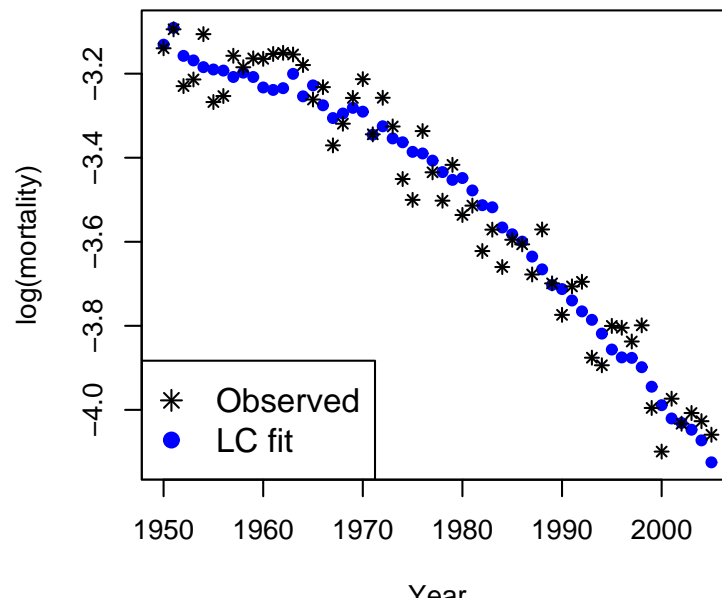
Original LC: Beta



Original LC: Kappa



Original LC: Age 70



Mix & match smoothing in LC

Discrete: α \leftrightarrow Smooth: $B_a a$

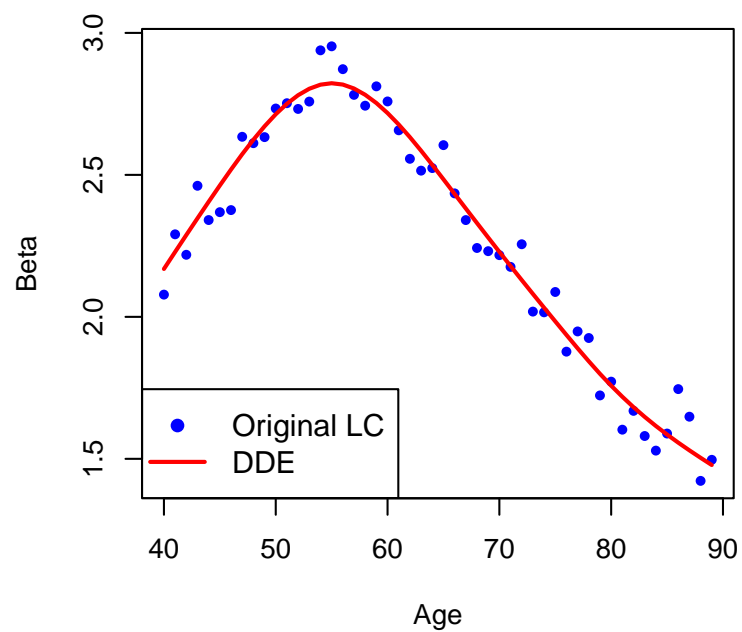
Discrete: β \leftrightarrow Smooth: $B_a b$

Discrete: κ \leftrightarrow Smooth: $B_y k$

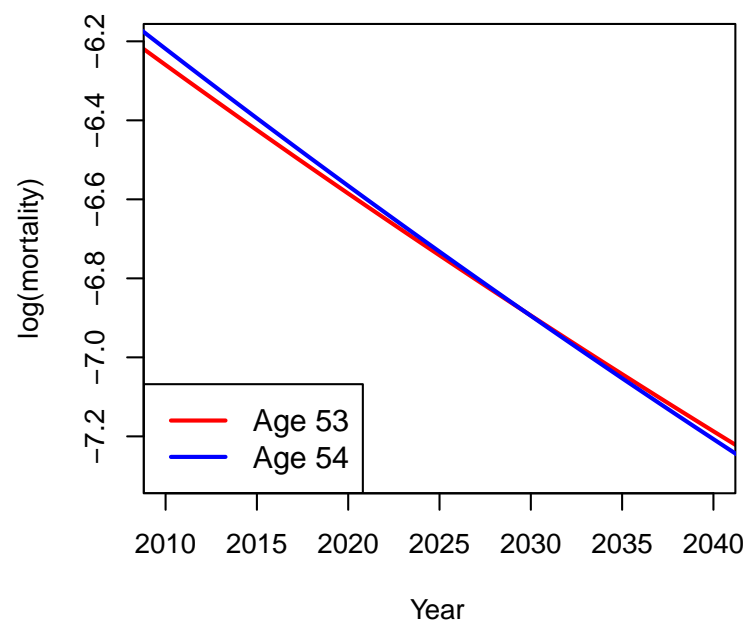
Delwarde, Denuit & Eilers (2007)

Smooth β and stabilize forecasts.

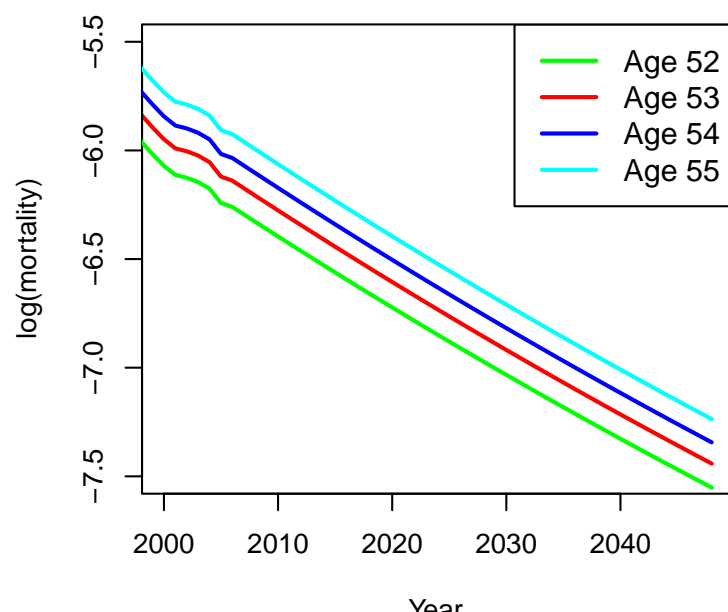
Beta in LC and DDE



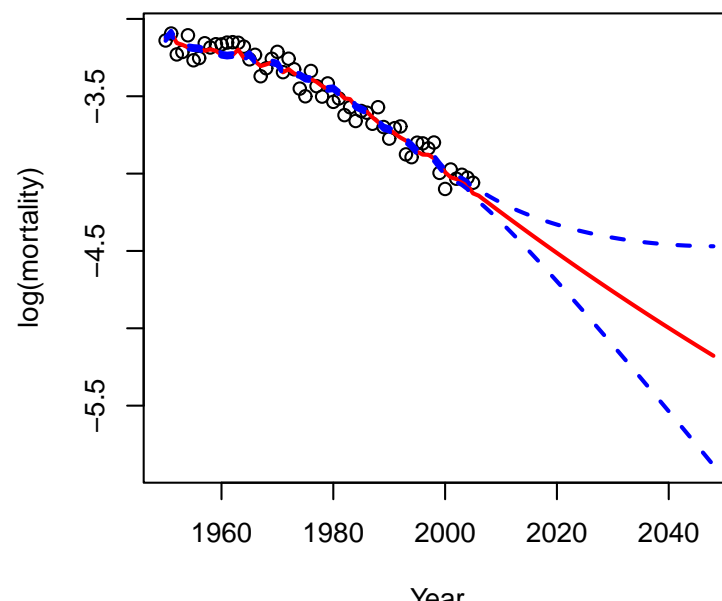
LC forecasts



DDE forecasts



DDE forecast with CIs for age 70



Richards & Currie (2009)

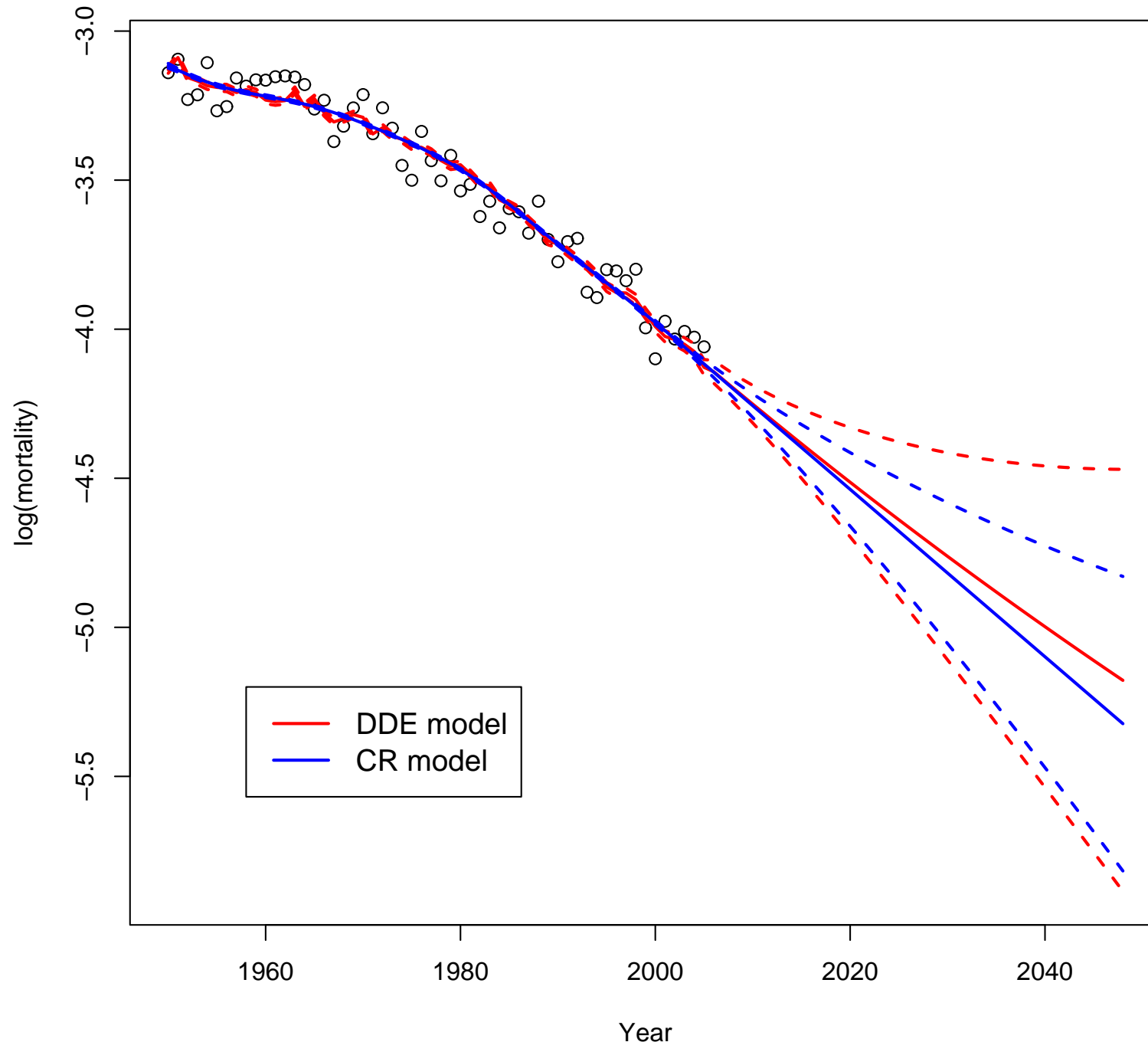
- Smooth β and stabilize forecasts
- Smooth κ and use penalty forecasts

$$\alpha \rightarrow \alpha$$

$$\beta \rightarrow B_a b$$

$$\kappa \rightarrow B_y k$$

Comparison of DDE and CR models for age 70



The Time Series Branch of the LC Family

Model	$(\mathbf{X}_\alpha, \mathbf{X}_\beta, \mathbf{X}_\kappa)$	$(\lambda_\alpha, \lambda_\beta, \lambda_\kappa)$
LC	$(\mathbf{I}_{n_a}, \mathbf{I}_{n_a}, \mathbf{I}_{n_y})$	$(0, 0, 0)$
DDE	$(\mathbf{I}_{n_a}, \mathbf{B}_a, \mathbf{I}_{n_y})$	$(0, \lambda_\beta, 0)$
SLC	$(\mathbf{B}_a, \mathbf{B}_a, \mathbf{I}_{n_y})$	$(\lambda_\alpha, \lambda_\beta, 0)$
GLC	$([\mathbf{1}, \mathbf{x}_a], \mathbf{B}_a, \mathbf{I}_{n_y})$	$(0, \lambda_\beta, 0)$

The Penalty Branch of the LC Family

Model	$(\mathbf{X}_\alpha, \mathbf{X}_\beta, \mathbf{X}_\kappa)$	$(\lambda_\alpha, \lambda_\beta, \lambda_\kappa)$
CR.P	$(\mathbf{I}_{n_a}, \mathbf{B}_a, \mathbf{B}_y)$	$(0, \lambda_\beta, \lambda_k)$
SLC.P	$(\mathbf{B}_a, \mathbf{B}_a, \mathbf{B}_y)$	$(\lambda_\alpha, \lambda_\beta, \lambda_\kappa)$
GLC.P	$([\mathbf{1}, \mathbf{x}_a], \mathbf{B}_a, \mathbf{B}_y)$	$(0, \lambda_\beta, \lambda_\kappa)$

References

- Brouhns, Denuit & Vermunt (2002) Insurance: Mathematics and Economics, 31, 373-393.
- Delwarde, Denuit & Eilers (2007) Statistical Modelling, 7, 29-48.
- Eilers & Marx (1996) Statistical Science, 11, 758-783.
- Lee & Carter (1992) Journal of the American Statistical Association, 87, 659-671.
- Richards & Currie (2009) British Actuarial Journal (to appear)
- IDC's web page: Talks in
`www.ma.hw.ac.uk/~iain/research/talks/talks.html`
and papers in
`www.ma.hw.ac.uk/~iain/research/talks/papers.html`
- Longevity blogs: See *The Lee-Carter Family (July 10, 2010)* in
`www.longevity.co.uk/site/informationmatrix/`