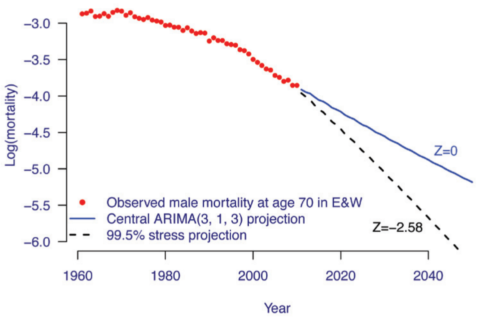
Value-at-risk (VaR) for longevity trend

LONGEVITY RISK

Longevity risk is the chance of a long-term adverse trend unfolding over many years. This risk is typically present in life-company annuity portfolios and defined-benefit pension schemes. Many insurers and other companies now need to closely manage their exposure to longevity risk.

TREND STRESS

The best way of exploring longevity risk is to use a stochastic projection model. This will give a central estimate of mortality trend, together with measures of uncertainty in the form of the standard error. We can get a stress scenario by subtracting the appropriate number of standard errors from the central projection.



We can calculate the capital required for longevity trend risk as the relative increase in the stressed annuity factor compared to the central estimate:

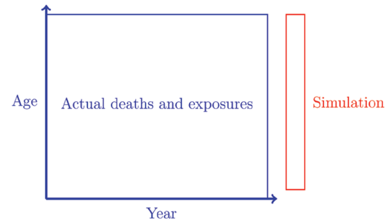
$$\frac{\bar{a}_x^{Z=-2.58}}{\bar{a}_x^{Z=0}} - 100\%$$

We can calculate the above capital requirement using a variety of standard models:

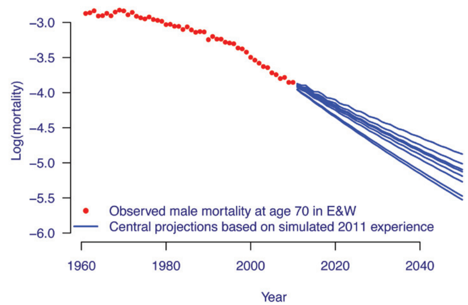
Projection model	Stress capital
Lee-Carter (1992)	6.28%
Cairns-Blake-Dowd (2006)	6.22%
Age-Period-Cohort (APC)	4.35%

SOLVENCY II AND VALUE-AT-RISK

The stressed-trend approach is a way of tackling longevity risk over the lifetime of the annuitant. However, a valid risk-management question is "by how much could the central estimate change based on an extra year's data?". In particular, the stressed-trend approach does not fit into the Solvency II requirement for risk viewed over a one-year time horizon. An alternative therefore is to simulate an extra year's experience and refit the model to see what impact an extra year's data can have on the central estimate.



This process of simulation and refitting can be repeated a thousand times (say) to get a set of simulated update scenarios.



These projections can then be used to calculate a set of annuity factors:

$$S = \{\bar{a}_x^j; j = 1, 2, \ldots, 1000\}$$

The value at risk over a one-year time horizon – or the capital for longevity trend risk under Solvency II – would then be:

$$\frac{99.5^{\text{th}} \text{ percentile of } S}{\text{mean of } S} - 100\%$$

MODEL RISK

It is critical not to rely on a single model, and longevity trend risk is no exception to this.

The framework outlined here is general enough to accommodate a wide variety of standard models. Here are some example figures using 1,000 simulations of the procedure for some well-known models:

Projection model	One-year capital
Lee-Carter (1992)	4.99%
Cairns-Blake-Dowd (2006)	3.91%
Age-Period-Cohort (APC)	2.96%

The figures for your portfolio will be different, as you will need specimen annuities which are representative of your actual liabilities. You may also like to try other stochastic models.

Of course, the figures here just apply to trend uncertainty, and there are more aspects to longevity risk than this. For more details see Richards, Currie and Ritchie (2012).

ENSURING ROBUSTNESS

The framework described here can be used with most mainstream stochastic projection models. Besides yielding answers to the value-at-risk question, repeated simulation and refitting also tests a given model's ability to cope with new data year-on-year. This gives early warning of models which could produce undesirable results after making the commitment to use them.

Models fitted by

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References

CAIRNS, A. J. G. and BLAKE, D. and DOWD, K. (2006) A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration, Journal of Risk and Insurance, 73, 687–718.

LEE, R. D. and CARTER, L. (1992) Modelling and forecasting the time series of US mortality, Journal of the American Statistical Association, 87, 659–671.

RICHARDS, S. J., CURRIE, I. D. and RITCHIE, G. P. (2012) A value-at-risk framework for longevity trend risk, Longevitas Ltd.

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